Deutscher Wetterdienst
Wetter und Klima aus einer Hand

Recent developments in Numerical Methods - report from ‚PDE's on the sphere 2012'

28th WGNE meeting
05-09 Nov. 2012, Toulouse

M. Baldauf (DWD)

'Partial differential equations on the sphere'

24-28 Sept. 2012, Cambridge
Isaac Newton Institute for Mathematical Sciences

http://www.newton.ac.uk/programmes/AMM/ammw02.html

Sessions:
- Horizontal session
- Vertical session
- Time session
- Advection session
- Adaptive meshing
- Test cases
- From Dynamical cores to Operational Models
- Novel optimisation techniques

J. Thuburn, C. Cotter (2012) SIAM JSC

mimetic properties of numerical schemes

- nonspurious vorticity by pressure gradient term
  → discrete analogon of $\nabla \times \nabla \phi = 0$
- kinetic energy conservation
  → discrete analogon of $\mathbf{v} \cdot \nabla \phi + \phi \nabla \cdot \mathbf{v} = \nabla \cdot (\mathbf{v} \phi)$
- no energy production by Coriolis terms
  → discrete analogon of $\mathbf{v} \cdot \underbrace{(\mathbf{k} \times \mathbf{v})}_{=\mathbf{v}^{\perp}} = 0.$

Theoretical analysis of these properties and their realisation even on unstructured, non-orthogonal grids is progressing.

J. Thuburn, C. Cotter (2012) SIAM JSC

- conservation of prognostic variable → finite volume discretization of

$$\frac{\partial \phi}{\partial t} + \nabla \mathbf{f} = 0$$

- Existence of steady geostrophic modes: $f\mathbf{k} \times \mathbf{v} + \nabla \Phi = 0$

  due to the shallow water vorticity-eq. $\frac{\partial \xi}{\partial t} + f \,\mathrm{div}\, \mathbf{v} = 0$

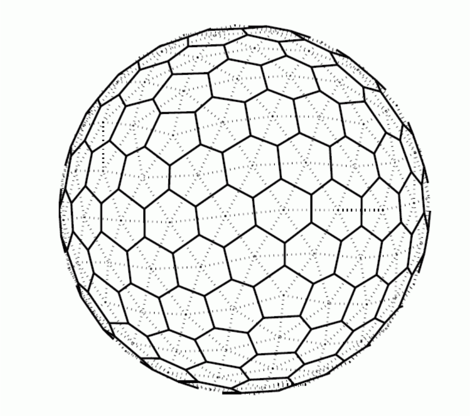
  the numerical divergence at the position of ξ must vanish!

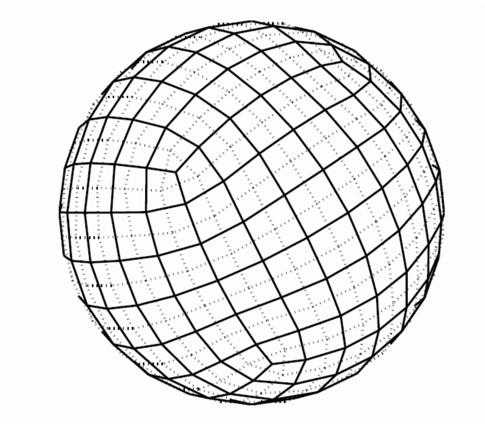
  not trivial (*Nickovicz et al., 2002)*
  solution on hexagonal C-grid by *Thuburn (2008), Thuburn, Ringler, Skamarock, Klemp (2009)*

In the recent years more and more **finite-element (FE) based methods** are upcoming in atmospheric modelling

- Finite elements (talk by C. Cotter)
- spectral elements (talks by M. Taylor, S. Gabersek)
- Continuous Galerkin
- Discontinuous Galerkin (talks by R. Nair, M. Kopera, A. Müller, G. Tumolo, ...)
  =FE + FV discretization

→ higher order methods reduce grid imprint in non-orthogonal
  or unstructured grids (icosahedral grids, cubed sphere)





(figures from talks J. Thuburn or T. Dubos)

... **FE-based methods**:

- computational intensity (= # of calculations : # of memory accesses) is high
  → advantages on new computer architectures (GPU's, ...) → scalability

- General problem in these FE-like schemes: small Courant number
  e.g. CFL ~ 1 / (1 + 2*p), p=Polynomial degree
  for the Discontinuous Galerkin (DG) Method

  → Combination with other schemes:
  Semi-Lagrangian-DG (talks by R. Nair, G. Tumolo)

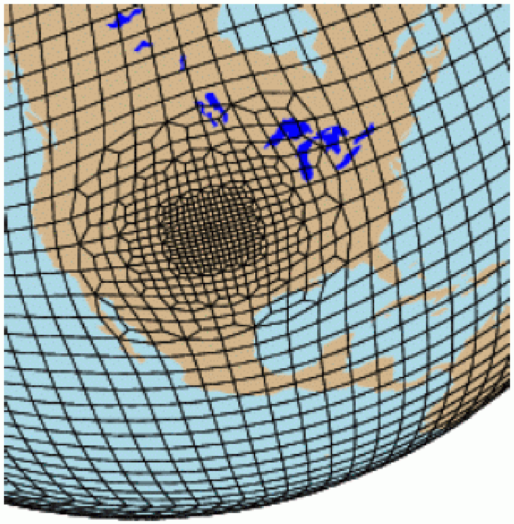
Spectral Element Method

M. Taylor

Sandia National Laboratories

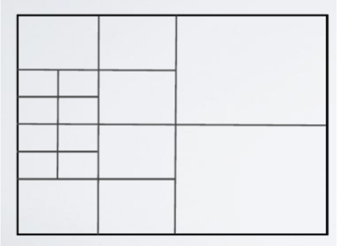
CAM-SE

- CG finite element method with quadrature-based inner-product (diagonal mass matrix)
- Simple Q^3-Q^3 element (velocity and pressure are both in Q^3), stabilized with hyper-viscosity
- Mimetic discretization (Taylor & Fournier JCP 2010):
    - Discretization preserves adjoint properties of DIV, GRAD and CURL operators:
        - local conservation of quantities in conservation form (mass, tracer mass)
        - Semi-discrete conservation of energy
    - CURL GRAD = 0
        - Local conservation of 2D PV
        - Stationary geostrophic modes (f-plane)



Adaptive meshing (=topic of a Newton-institute program)

- Tropical cyclone (hurrican) *global* simulation with variable grids using CAM-SE (cubed-sphere, spectral elements) with 2° … 0.5° (0.25°) grid mesh size with *conformal* grid refinement in the vicinity of the cyclone (*C. Zarzicki, Univ. Michigan*)
- Adaptive Discontinuous Galerkin simulations (*A. Müller, Univ. Mainz*)
- Proposal for flux calculation for Discontinuous Galerkin on quadrilateral grid with *non-conformal* refinement (*M. Kopera, Naval Postgr. School*)



Generally: improvements of refinement numerical techniques,
but no improvement in refinement/coarsening criteria visible.
still tropical cyclone is only possible application for NWP/climate simul.

Time session

- General analysis of multi-level semi-implicit schemes with two frequencies =fast & slow modes (*C. Clancy*)

- Laplace-transform method (*P. Lynch, Univ. Dublin)*
  to solve the fast waves
  part (approximately linear) directly, time filtering possible by excluding poles in the complex s-plane
  phase errors are much smaller than for semi-implicit scheme

- time parallel scheme (*J. Cote, Univ. Quebec)*

Test cases

- reports form the DCMIP workshop (*C. Jablonowski (Univ. Michigan), P. Lauritzen (NCAR)*)
- Evaluation of Models by using asymptotic limit solutions (*M. Cullen, UK MetOffice*)
- An analytic solution for the compressible Euler-equations on the sphere (*M. Baldauf, DWD*)

C. Jablonowski
(Univ. Michigan)

Highlights of the Dynamical Core Model Intercomparison Project (DCMIP)

Christiane Jablonowski, Paul Ullrich, James Kent, Kevin Reed, Mark Taylor, Peter Lauritzen, Ram Nair

& Jennifer Williamson (NCAR admin support)

The Ideas behind DCMIP

C. Jablonowski (Univ. Michigan)

- The 2-week summer school and model intercomparison project DCMIP-2012 highlighted the newest modeling techniques for global climate and weather models
- Took place at NCAR from July/30-August/10/2012
- Brought together over 26 modeling mentors and organizers, 37 students, and 19 speakers
- DCMIP-2012 paid special attention to non-hydrostatic global models and their dynamical cores that now emerge in the GCM community
- Hosted 18 participating dynamical cores (3 participated remotely)

The Ideas behind DCMIP

C. Jablonowski (Univ. Michigan)

- DCMIP-2012
  - Taught students, postdocs and the GCM community, both via lectures and hands-on sessions, at NCAR and elsewhere in the world (via the webcast and recordings): http://earthsystemcog.org/projects/dcmip-2012/lectures
  - Conducted an international dynamical core model intercomparison
  - Defined, tested and probably establishes new dynamical core tests
- Our vision: establish DCMIP as a long-term virtual community via the cyberinfrastructure-supported workspace
- Gateway to the virtual community, and open invitation to become a member and participate:

http://earthsystemcog.org/projects/dcmip-2012/

The Architecture of the DCMIP Test Suite

The tests are hierarchical and increase in complexity

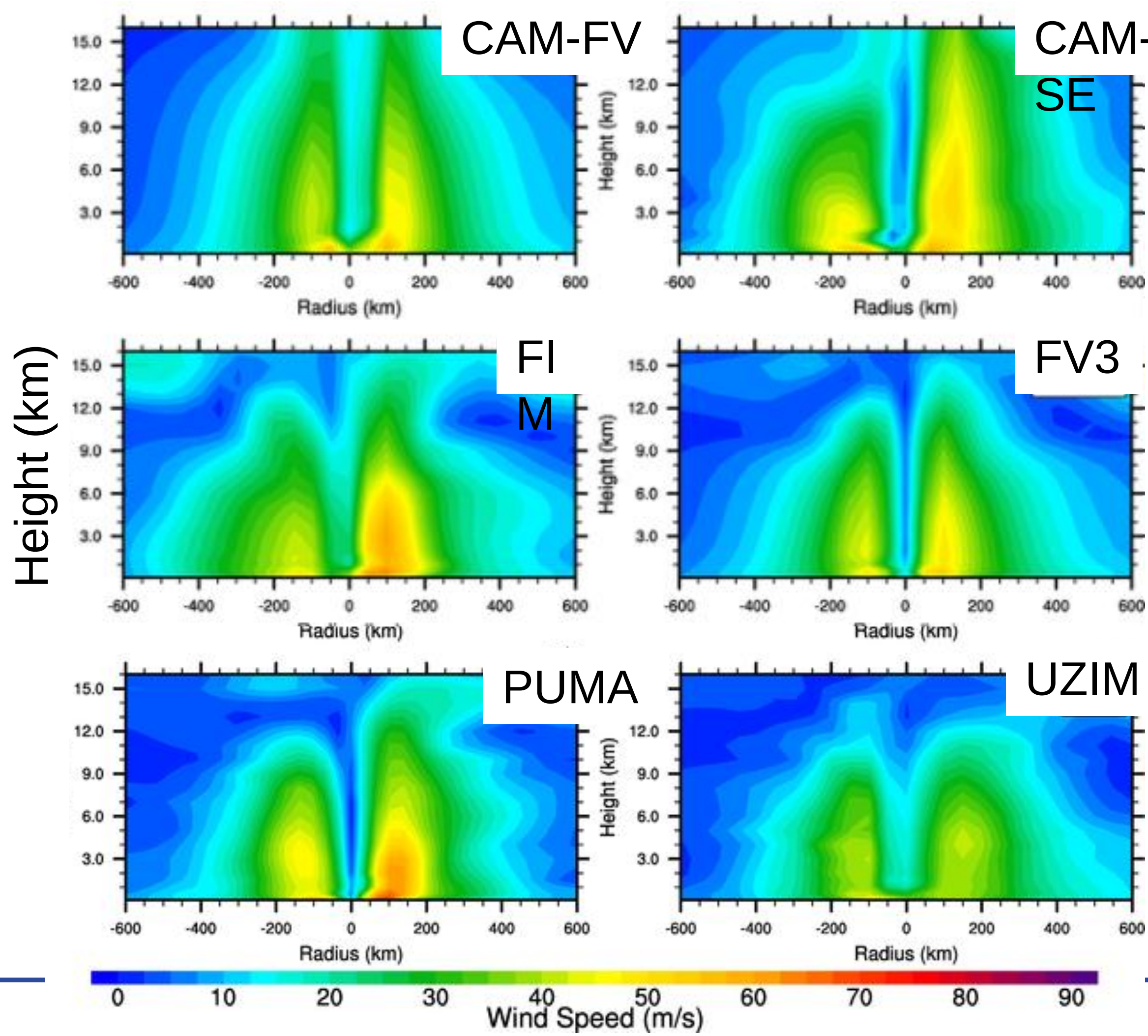
C. Jablonowski (Univ. Michigan)

http://earthsystemcog.org/projects/dcmip-2012/test_cases

- **3D Advection**
  - Pure 3D advection without orography
  - Pure 3D advection in the presence of orography
- ➔ **Dry dynamical core without rotation**
  - Stability of a steady-state at rest in presence of a mountain
  - Mountain-induced gravity waves on small planets
  - Thermally induced gravity waves on small planets

- ➔ **Dry dynamical core with the Earth's rotation**
  - From large (hydrostatic) to small (nonhydrostatic) scales, nonlinear baroclinic waves on a shrinking planet with dynamic tracers PV and θ
- **Simple moisture feedbacks**
  - Moist baroclinic waves with large-scale condensation
  - Moist baroclinic waves with simplified physics (simple-physics)
  - Idealized tropical cyclones

Tropical Cyclone with Simple-Physics



Test 51: Idealized TC on an aqua-planet: Simulations with Simple-Physics

Height-longitude cross section of the wind speed (m/s) at day 10: wide spread in results

0.5° x 0.5° L30, dx= 55 km

Reed and Jablonowski (MWR, 2011)
Reed and Jablonowski (James, 2012)

Some Final Observations

- We see both very interesting agreements and spreads among the results, let us learn from them
- The spread will give us insight into the uncertainty of the simulations, and the characteristics of the numerical schemes
- We might find bugs in our codes, and DCMIP data
- We might want to fine-tune the test cases
- DCMIP-2012 is not a ‘beauty contest’: our goal is to expose the pros and cons of the dynamical core modeling approaches via objective evaluations

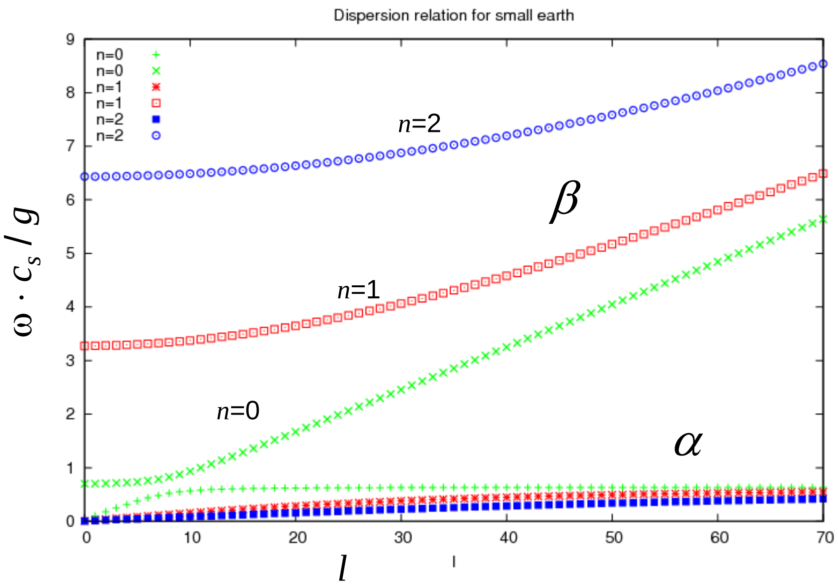
Analytic solution for the compressible Euler equations on the sphere

for the vertical velocity w (Fourier component with k_z, spherical harmonic with l,m)

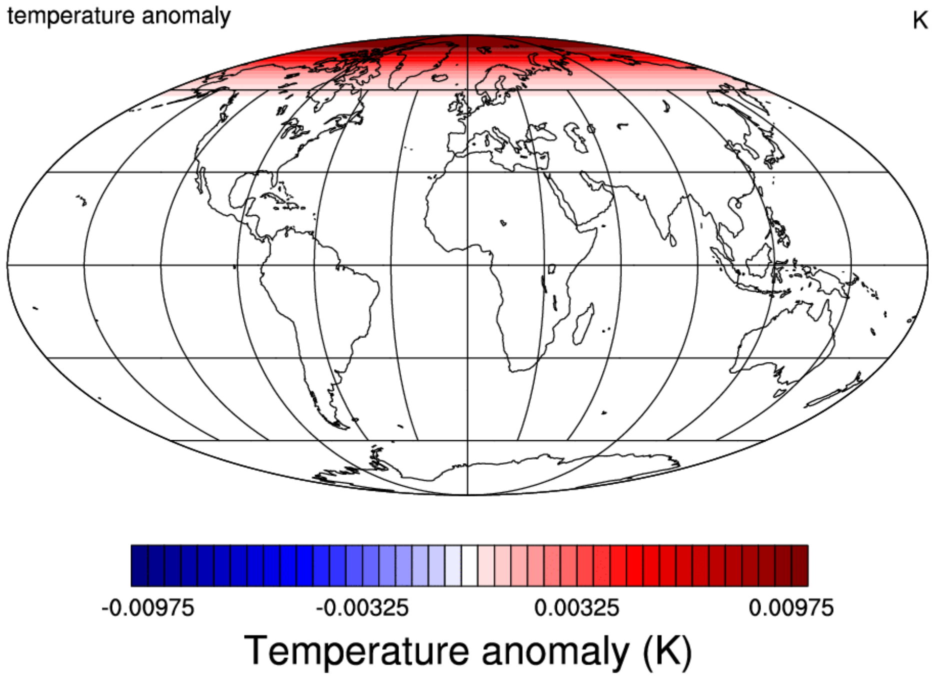
$$\hat{w}_{lm}(k_z,t) = -\frac{1}{\beta^2-\alpha^2}\left[-\alpha\sin\alpha t + \beta\sin\beta t + \left(f^2 + c_s^2\frac{l(l+1)}{r_s^2}\right)\left(\frac{1}{\alpha}\sin\alpha t - \frac{1}{\beta}\sin\beta t\right)\right] g\frac{\hat{\rho}_{lm}(k_z,t=0)}{\rho_s}$$

analogous expressions for $\hat{u}_{lm}(k_z, t)$, ...

The frequencies α, β are the gravity wave and acoustic branch, respectively, of the dispersion relation for compressible waves in a spherical channel of height H; $k_z = (\pi / H) \cdot n$



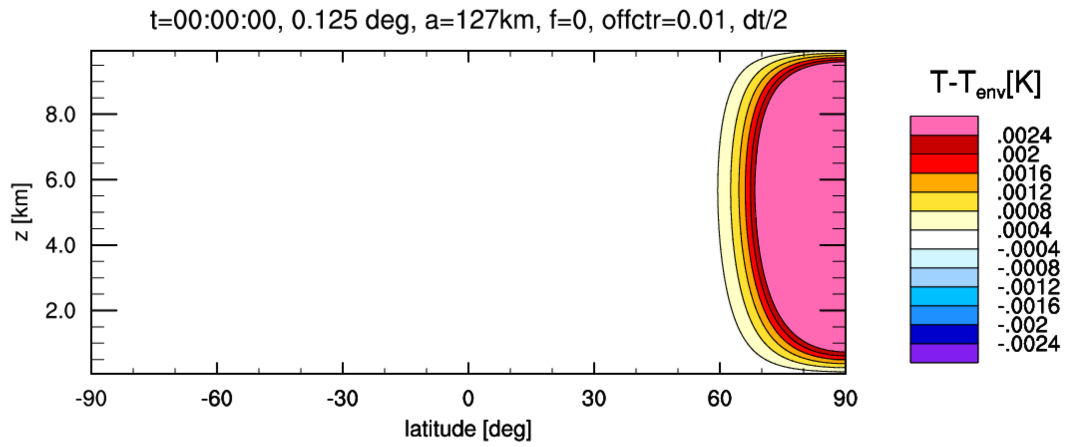
ICON simulation, *f*=0



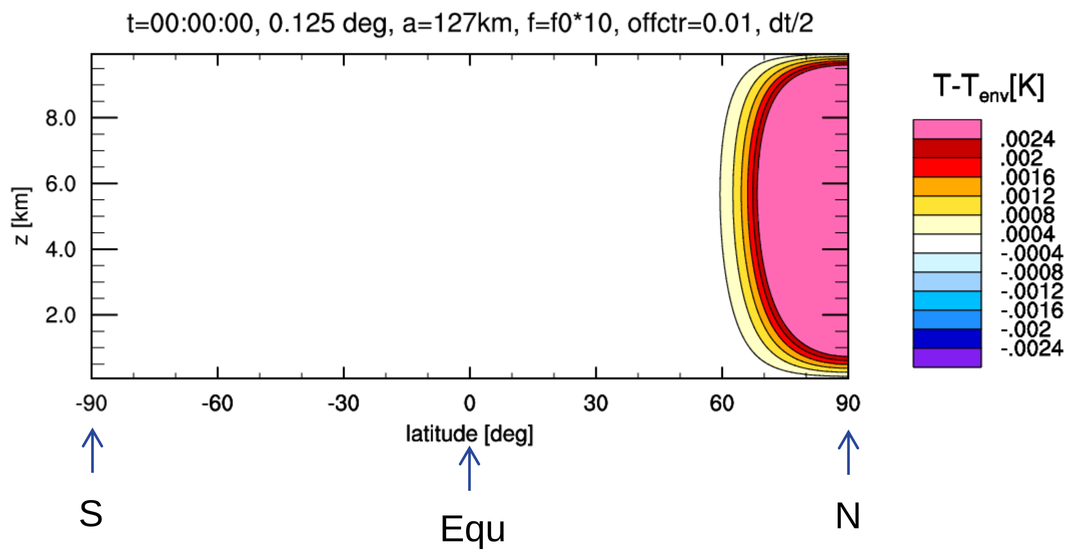
in z=5 km

Time evolution of T‘

f=0



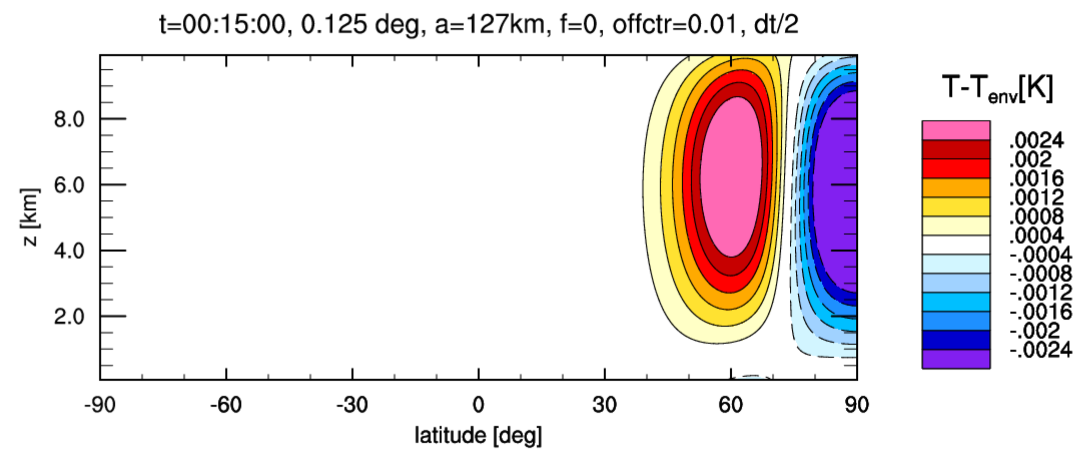
f≠0



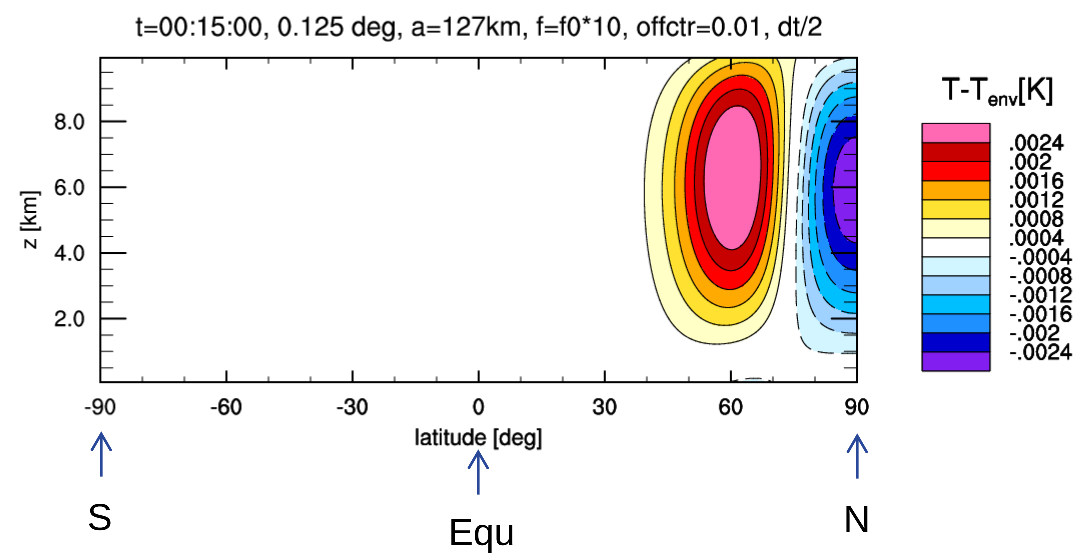
Solid lines: analytic solution
Colours: ICON simulation

Time evolution of T‘

f=0



f≠0



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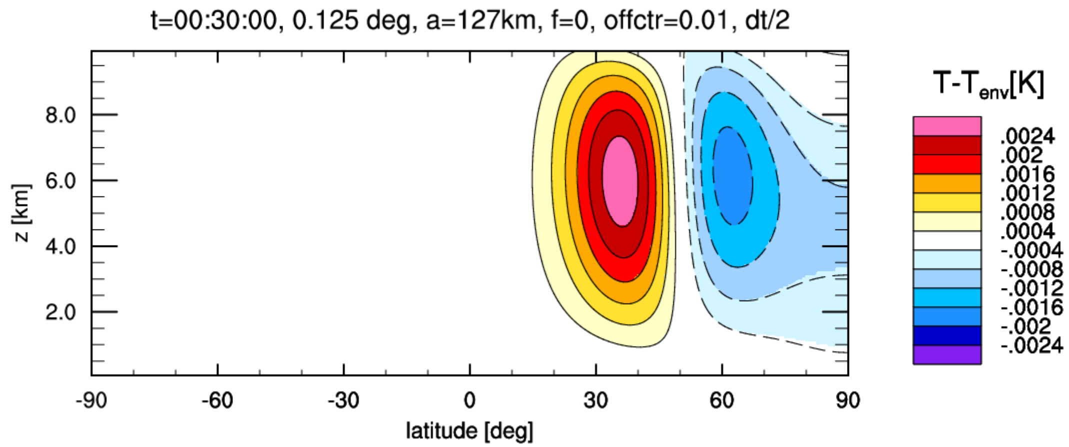
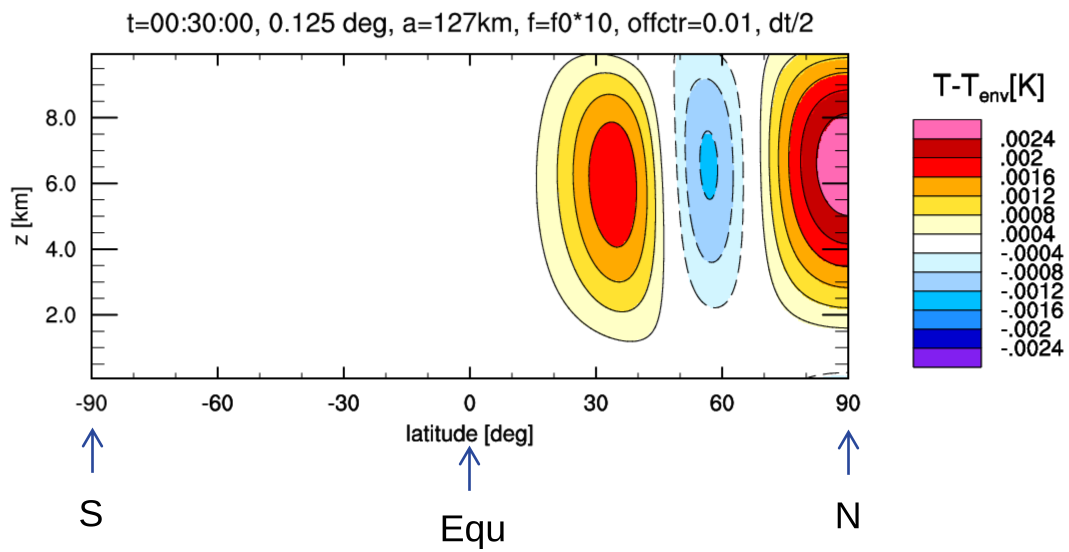
Time evolution of T‘

$f=0$



$f\neq0$



Solid lines: analytic solution
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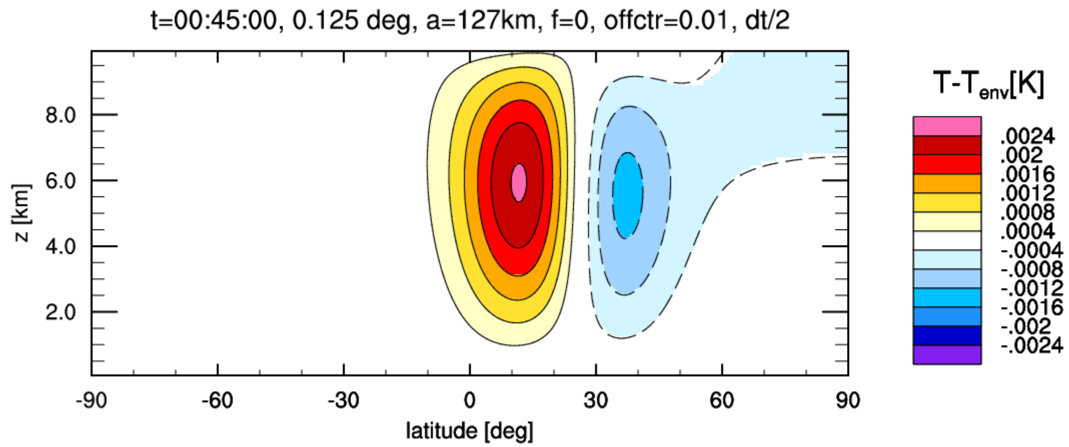
Time evolution of T‘

$f=0$



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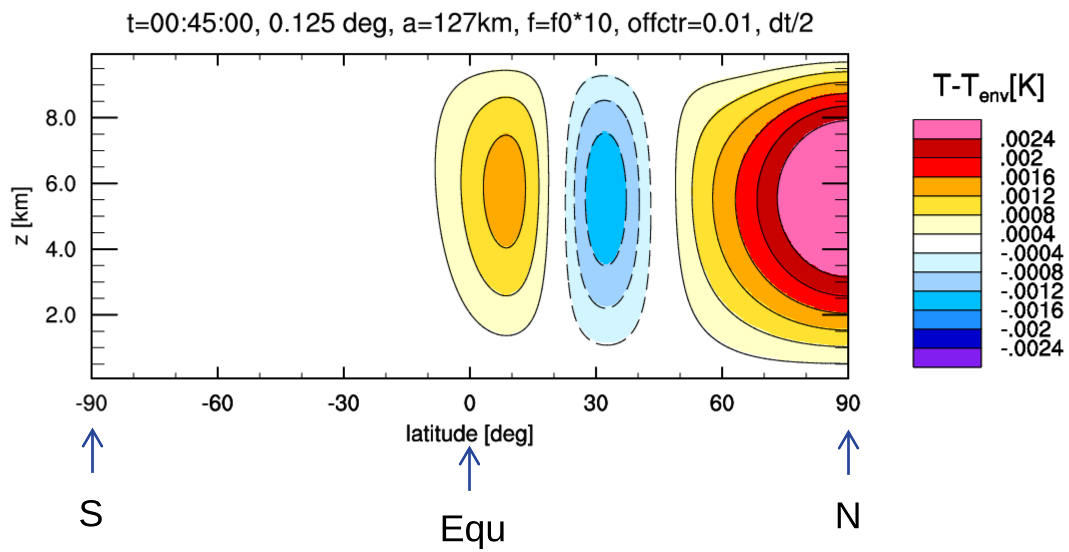
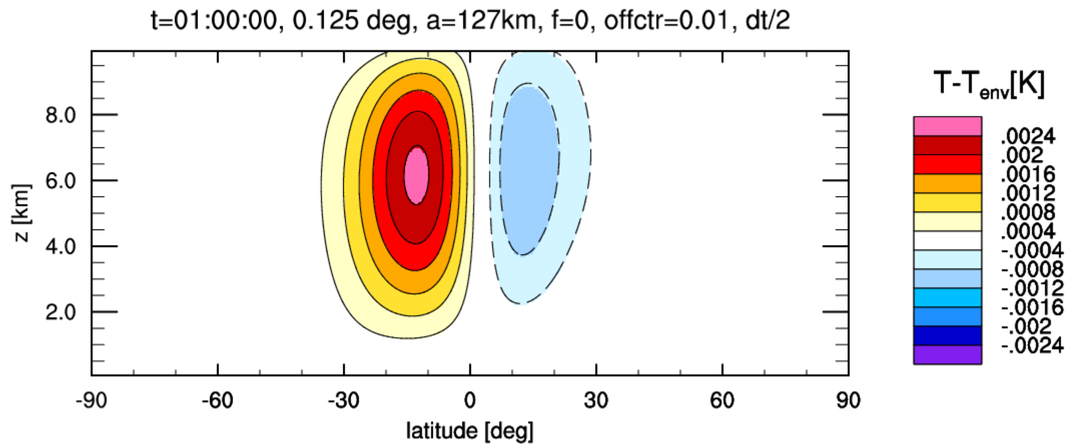


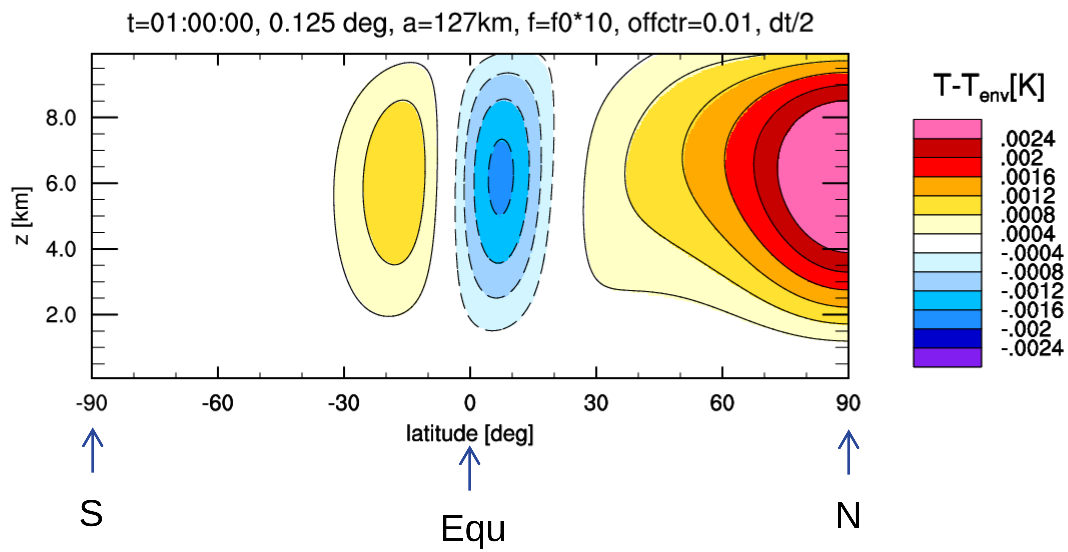
Solid lines: analytic solution
Colours: ICON simulation

Time evolution of T‘

f=0



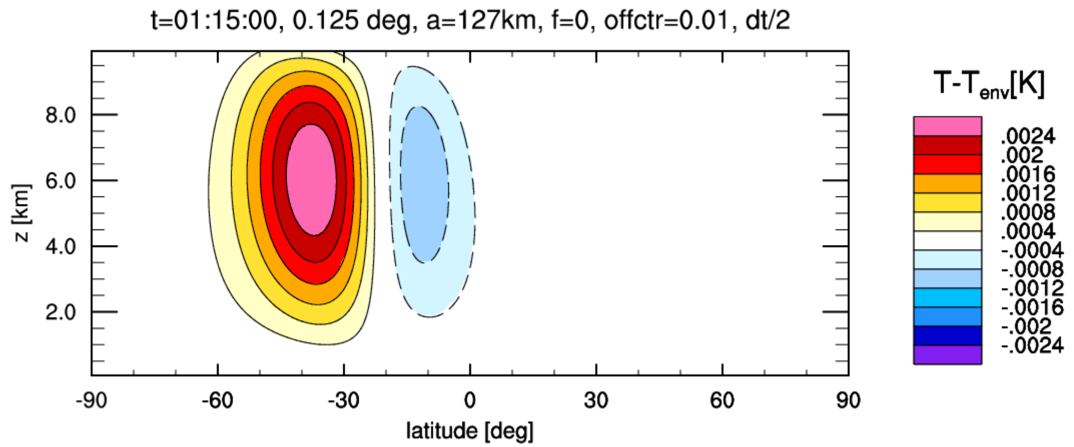
f≠0



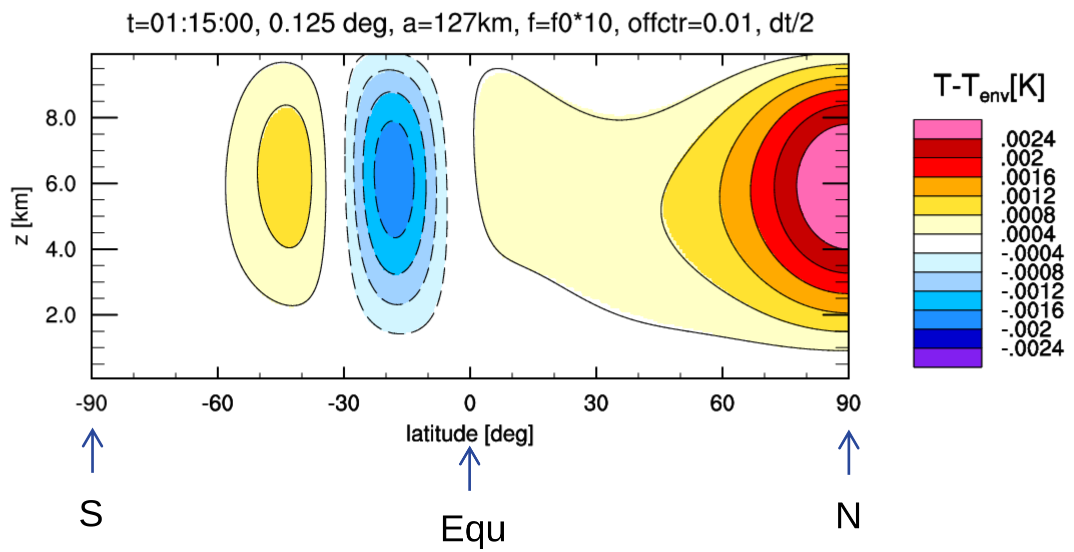
Solid lines: analytic solution
Colours: ICON simulation

Time evolution of T'

f=0



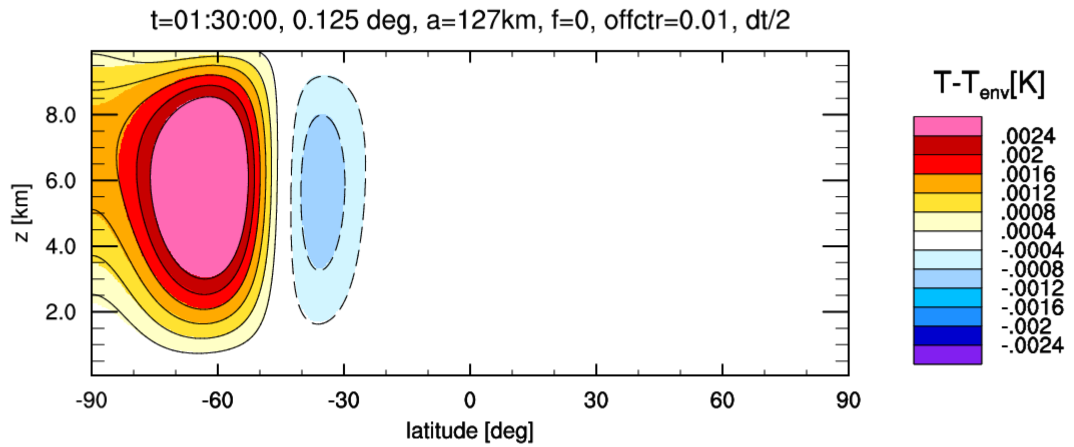
f≠0



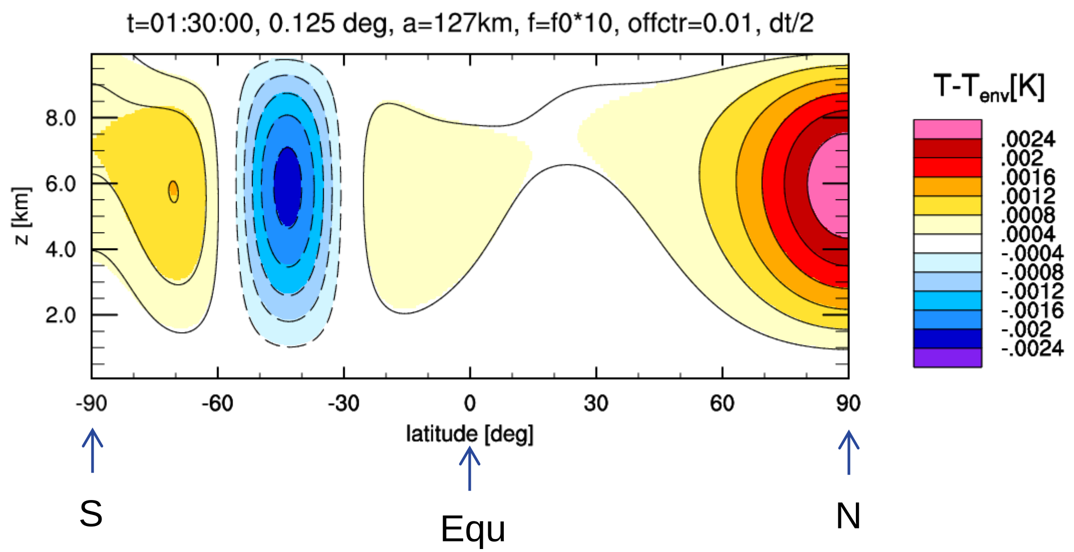
Solid lines: analytic solution
Colours: ICON simulation

Time evolution of T‘

f=0



f≠0

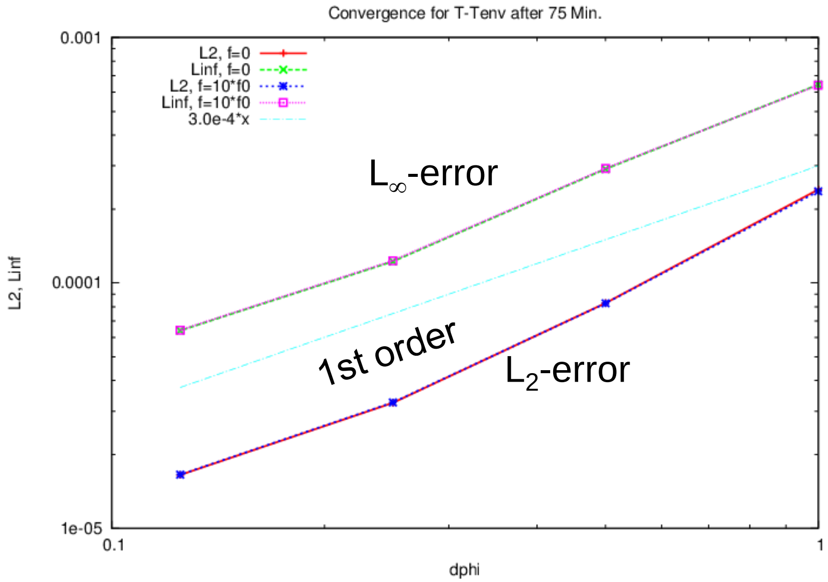


Solid lines: analytic solution
Colours: ICON simulation

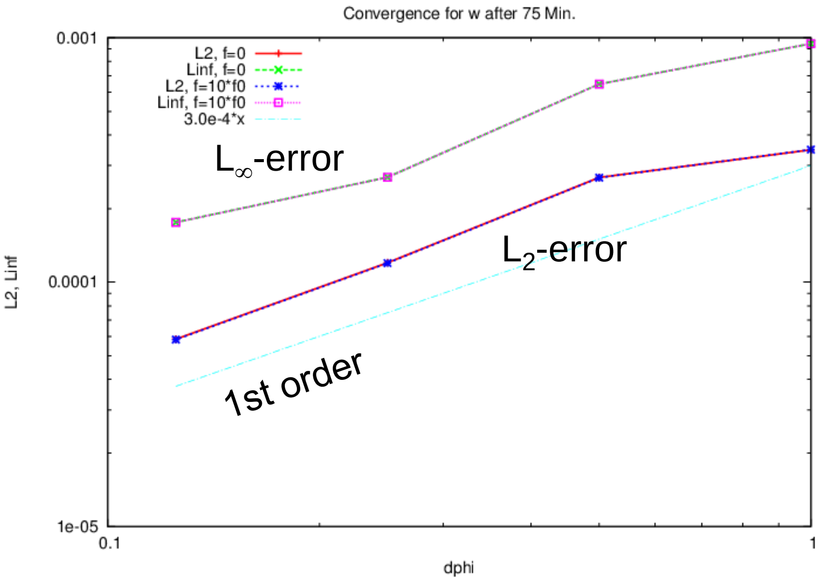
Convergence rate of the ICON model

- The ICON simulation with/without Coriolis force produces almost similar L_2, L_∞ errors
- L_2, L_∞ errors for w are generally higher than for T‘
- convergence order of ICON is ~ 1

T‘



w‘



From dynamical cores to operational models

ENDGame (Even newer Dynamics ...)

not changed:

- equation set & variables (θ-π)
- Arakawa-C + Charney-Philips-staggering
- SI-SL scheme

changes made:

- time integration:
  more implicit terms, but reduced off-centering
  'red/black' preconditioner → greatly reduced communications
- Advection: same SL scheme for all variables → improved robustness
- Coriolis: based on mass flux var. → improved Rossby mode propagation
- grid: V-at-poles → not solving Helmhotz problem at a singular point

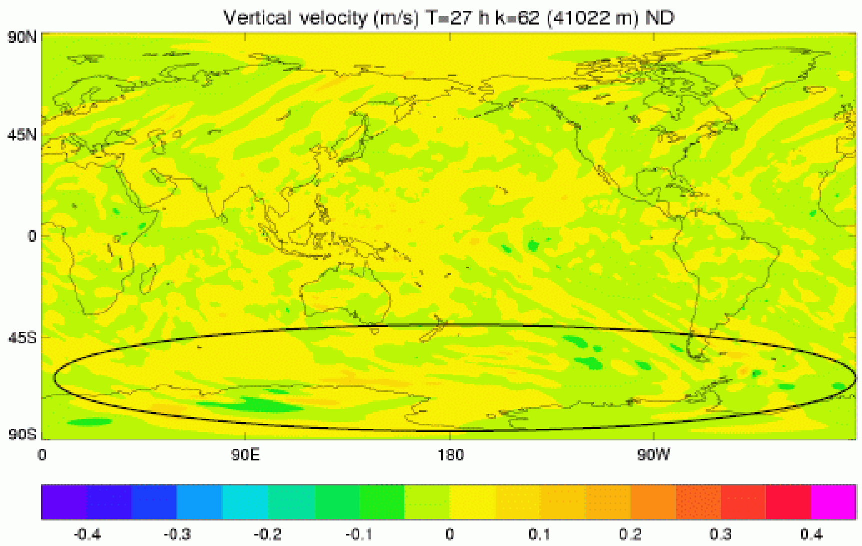
T. Melvin, N. Wood (UK MetOffice)

Met Office

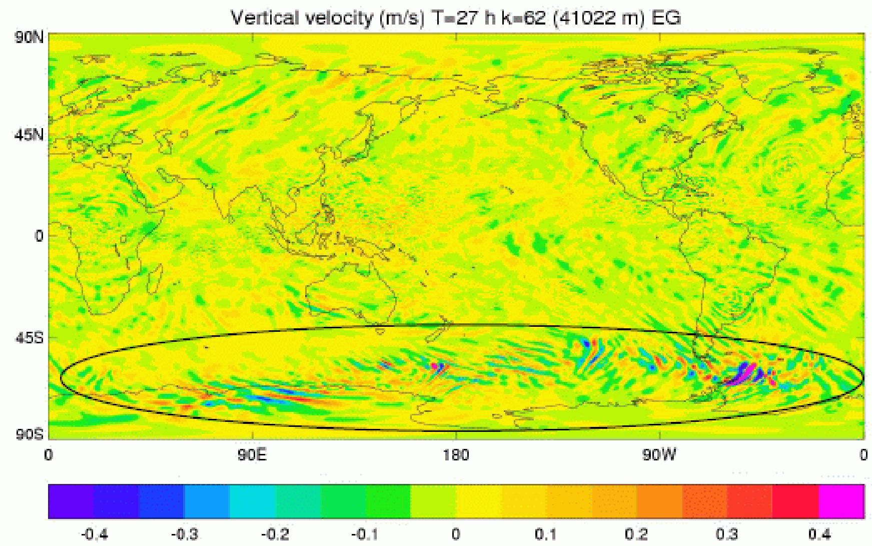
Accuracy: Stratospheric winds

UNIVERSITY OF EXETER

Vertical velocity at 41km N320



New Dynamics



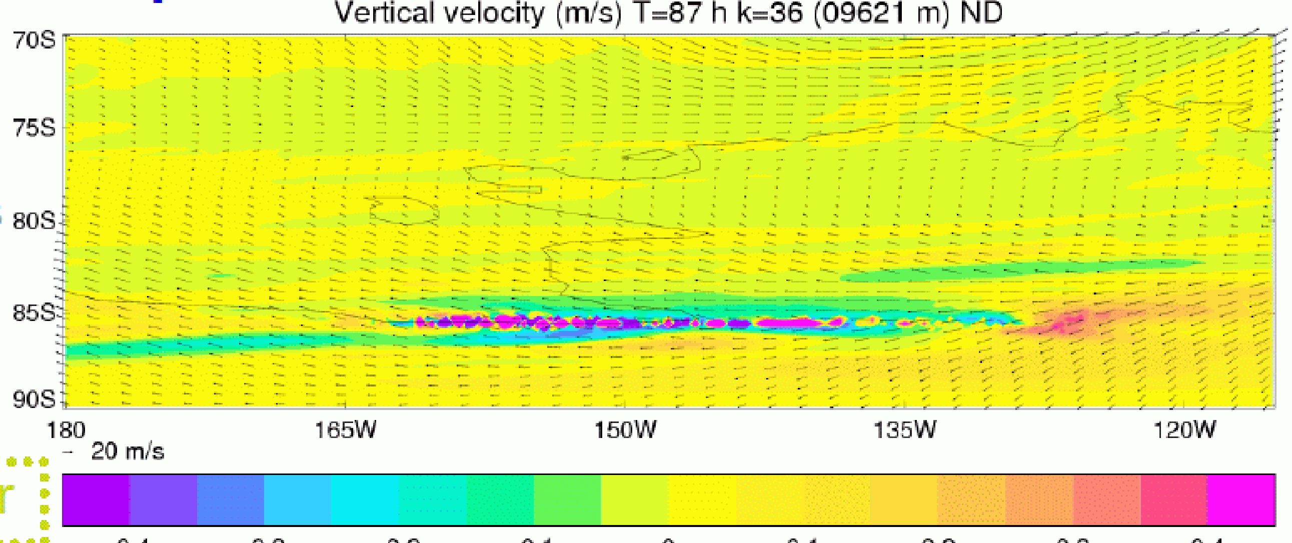
ENDGame

Simon Vosper

T. Melvin, N. Wood (UK MetOffice)

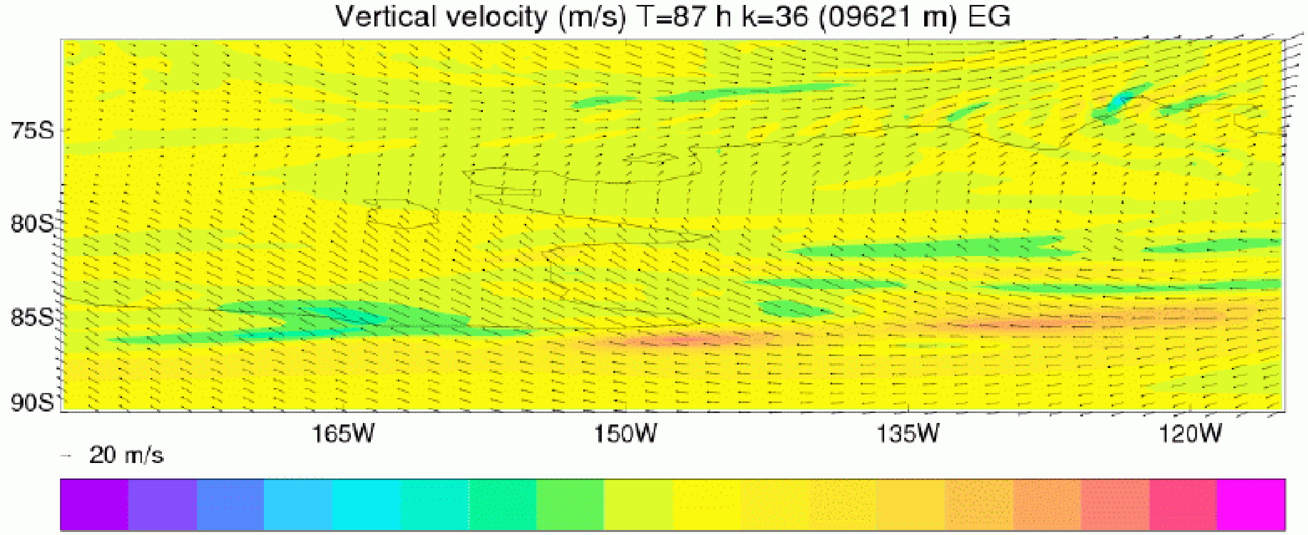
Stability: September Antarctic failures

New Dynamics

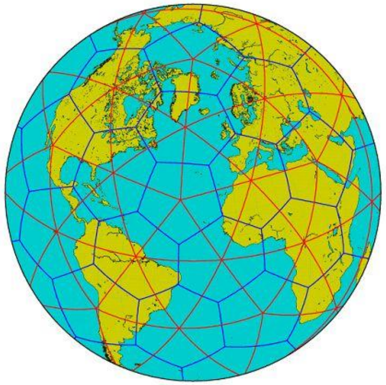


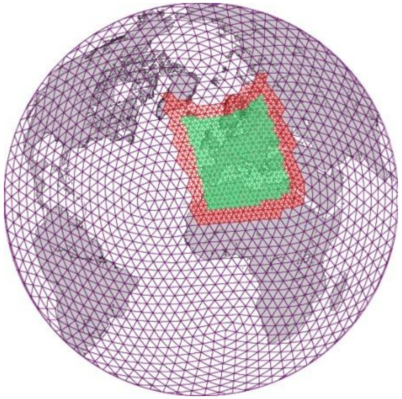
Simon Vosper

ENDGame



G. Zängl (DWD)





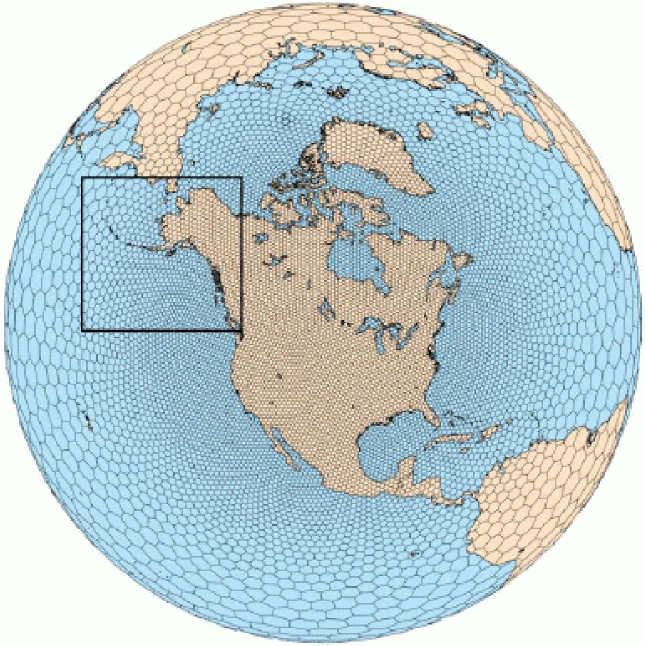
ICON = ICOsahedral Nonhydrostatic model

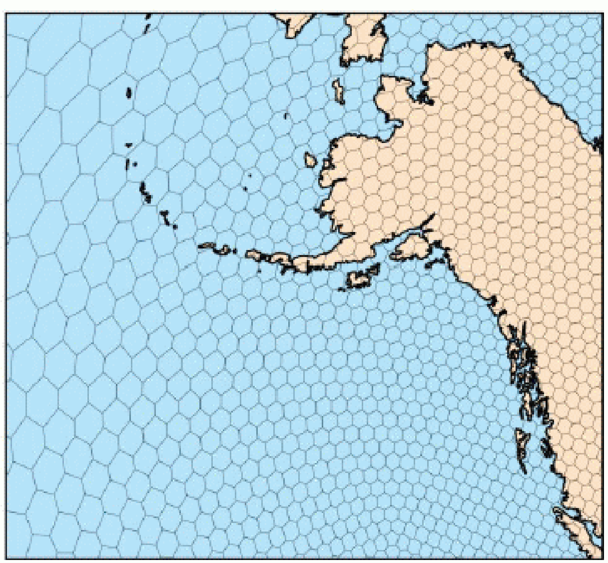
→ center report DWD

- **Joint development project of DWD and Max-Planck-Institute for Meteorology for the next-generation global NWP and climate modeling system**
- **Nonhydrostatic dynamical core on an icosahedral-triangular C-grid; coupled with (almost) full set of physics parameterizations**
- **Two-way nesting with capability for multiple nests per nesting level; vertical nesting, one-way nesting mode and limited-area mode are also available**

MPAS: C-Grid Spherical Centroidal Voronoi Meshes

B. Skamarock (NCAR)





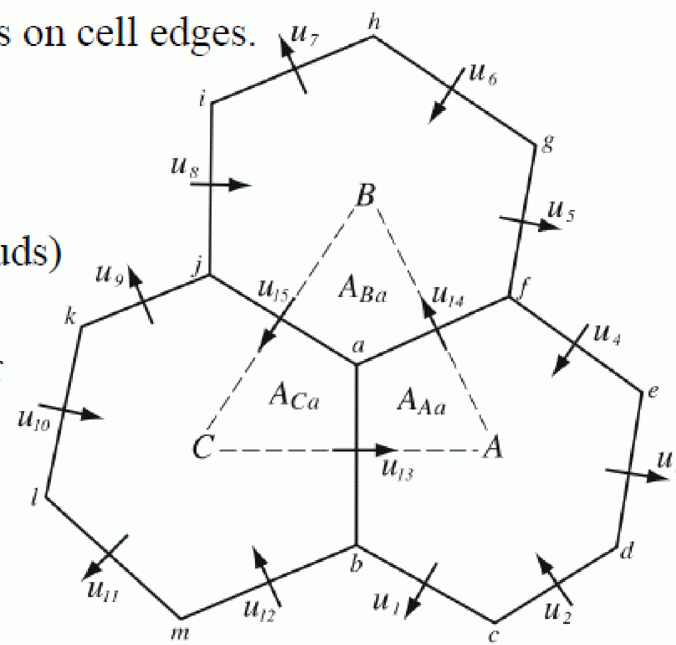
Unstructured mesh
- Mesh generation uses a density function.
- Uniform resolution – traditional icosahedral mesh.

Centroidal Voronoi
- Mostly *hexagons*, some pentagons and 7-sided cells.
- Cell centers are at cell center-of-mass.
- Lines connecting cell centers intersect cell edges at right angles.
- Lines connecting cell centers are bisected by cell edge.

C-grid
- Solve for normal velocities on cell edges.

Equations
- Fully compressible nonhydrostatic equations (*explicit* simulation of clouds)

Solver Technology
- Integration scheme similar to WRF.
- WRF-NRCM physics



Filters in MPAS

B. Skamarock (NCAR)

(1) Damping from the time-integration scheme.
(2) APVM.
(3) Upwinding in the transport scheme.
(4) Filtering in the model parameterizations (mostly vertical).
(5) Gravity-wave absorbing layer (Rayleigh damping – w)
(6) 2D diffusion and hyper-diffusion

$$K_2 \nabla_\zeta^2 \phi = K_2 \left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} \right)$$

$$K_4 \nabla_\zeta^2 \left(\nabla_\zeta^2 \phi \right) = K_4 \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} \right)^2$$

(7) Smagorinsky: $K_h = c_s^2 l^2 |Def|$

$$K_h \nabla_\zeta^2 \phi = K_h \left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} \right)$$

Current configurations use Smagorinsky; $K_2 = K_4 = 0$.

B. Skamarock (NCAR)

MPAS Variable-Resolution Mesh Tests Summary

Nonhydrostatic MPAS - hydrostatic flow regime, global-synoptic-mesoscale

Idealized case – normal mode initialized baroclinic-wave simulations.

Forecasts – October 2010 tropical and extra-tropical cyclone cases.

<u>Explicit filtering – Smagorinsky-only</u>

Variable-resolution mesh solutions show noise not present in uniform-mesh solutions

(1) noise is most noticeable primarily in w and vorticity at upper levels

(2) noise attribution in uncertain (numerics, wave reflection/refraction)

<u>Explicit filtering – Smagorinsky and/or 4th-order hyperdiffusion</u>

Small amount of hyperdiffusion on momentum, theta, remove the noise

(small relative to that used in current operational climate/weather models).

- Other parameterizations (physics) need attention.
- Future: continued testing, hydrostatic – nonhydrostatic regime mesh transition.