

# Calibration of the Multigrid Beta Filter for Application in GSI and JEDI

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The Multigrid Beta Filter (MGBF), an efficient and scalable covariance operator, is being incorporated in the data assimilation systems used at NOAA's Modeling Center, the Grid-point Statistical Interpolation (GSI) and Joint Effort for Data Assimilation (JEDI), where it will replace the Recursive Filter (RF) (e.g., see Purser et al. 2003 for an isotropic application in GSI). On massively parallel computers, the computational domain is decomposed and parallelized horizontally. For that reason, a sequential operator, such as a RF, though by itself is very efficient, has difficulties exploiting computational parallelism in modeling horizontal covariances with large horizontal spread over several computational domains. The MGBF technique, which solves this problem by application of a Beta filter with a compact support applied through a multigrid paradigm, demonstrated a superior scaling in all preliminary tests (e.g., Purser et al., 2022, Rancic et al., 2023).

Performance of the MGBF in the homogeneous and isotropic case that we consider for now depends on several parameters. The main version of the filter, which applies the 2D version of the beta radial filter horizontally, followed by a swipe of a 1D beta filter applied in vertical directions, relies on parameters of the horizontal and vertical aspect tensors (in the isotropic version only their intensities,  $a_h$  and  $a_v$ ), and scale weights applied to each multigrid generation  $w_i, i = 1, 2, \dots, n$ , where  $nn$  is the number of generations. Thus, this set of parameters needs to be predefined for a successful performance of the filter. We use error statistics generated by the NMC method as a reference for calibration, from which we get typical horizontal and vertical correlation lengths ( $\lambda_h, \lambda_v$ ) for the various atmospheric fields. Technically, the task of calibration is to find multigrid parameters:

$$a_h, a_v, w_i = f(\lambda_h, \lambda_v)$$

Clearly, this problem does not have a unique solution. Thus, for now we adopted an approach which, though it does not guarantee the best combination of parameters, provides a set that will lead to normalized covariances of the required size in all directions. To this end, we assume a preliminary set a scale weights which gives a reasonable height of the resulting covariances, and then run a series of tests in a standalone version of the MGBF code in which we prescribe forcing by a unit impulse and vary only intensities of aspect tensors. On the output, we measure the amplitude of the resulting covariance,  $\psi_{max}$ , and the resulting correlation lengths, ( $l_h, l_v$ ). In such a constellation, the derived correlation lengths are only functions of the intensities of aspect tensors,  $a_h$  and  $a_v$ . When the initial predefined set of scale weights is divided by the maximum of the resulting covariance,  $\psi_{max}$ , in a repeated test we derive a normalized covariance of the same size. This method is for now applied in a tabular fashion, where input correlation lengths from the NMC method, ( $\lambda_h, \lambda_v$ ), are compared with the normalized covariances of the length ( $l_h, l_v$ ), produced by the MGBF in standalone tests, in order find correct values of aspect tensors ( $a_h, a_v$ ) that need to be used in GSI. In future, we plan to apply a machine learning (ML) version

of this algorithm, in which an artificial neural net (ANN) will be applied instead of lookup tables to accomplish the same task.

Along the way we have found that vertical covariances may have unexpectedly long correlation lengths. To avoid possible issues with the efficiency which that could cause, we have developed a new flavor of the MGBF, in which the multigrid logistics is applied not only in the horizontal, but both in horizontal and vertical directions, as schematically illustrated in Fig. 1. Technically, we consistently reduce vertical resolution at each successive grid generation, which systematically eliminates the need for large vertical filter swapes, in terms of number of grid spaces, at generations with higher resolutions.

Initially, MGBF was developed having in mind only modeling of the background error. However, new possible applications are emerging (e.g., localization of ensemble, analysis error, scale selection, etc.). Thus, an object-oriented version of the MGBF code is being developed, which is primary targeting application in the framework of the new data assimilation system, JEDI.



**Fig. 1.** On the left, schematically presented standard MGBF paradigm, in which vertical resolutions of all grid generations are treated equally. On the right, the latest MGBF version, in which vertical resolutions of successive grid generations are systematically reduced (halved). The thick grid boxes represent processors; G1, G2, and G3 denote successive grid generations 1, 2 and 3.

## References

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