Inserting Intermediate Generations in a Multigrid Beta Filter using Offset Diagonal Grids and Characterizing the Analysis Error

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1. Multigrid beta filter

The multigrid beta filter (MGBF) is being developed at NOAA's Environmental Modeling Center as a method for simulating the covariance operators used in data analysis or assimilation. It is expected initially to replace the recursive filter (Wu et al.) covariance scheme in the Real-Time Mesoscale Analysis (RTMA, see de Pondeca et al. 2011) on a limited-area rectangular domain, but a version is also being developed for the global domain using gnomonic cubed-sphere grids, each similar to the one used by the global FV3 model, but in a hierarchy of successively coarser scales for simultaneous parallel processing of additive quasi-Gaussian contributions to the final covariance. The advantage of the MGBF scheme over the recursive filter scheme is that it is capable of being made highly parallelizable since each of the quasi-Gaussian contributions are of finite support (unlike the recursive filters), which leads to relatively simple spatial decompositions into subdomains that can be handled on separate processors at the same time.

2. Diagonal offset grids

We are looking at ways to enhance the versatility of the MGBF scheme by introducing intermediate diagonally aligned horizontal grids in the multigrid hierarchy. In the basic hierarchy of "filtering grids" (on which we apply the quasi-Gaussian beta filters themselves), the progression of successively coarser grids involves a horizontal scale change of exactly two at each step up the hierarchy. Since the horizontal grids are square in the limited domain and on the cubed sphere, it is possible in both cases to expand the hierarchy by interleaving a series of diagonal grids that reduce this jump in scale to a smaller factor of sqrt{2}, and thereby expand the range of covariance profile shapes that a judiciously weighted hierarchy of the quasi-Gaussians at each scale can simulate.

If we arrange for the grid points in each subdomain to be offset a half grid space from its boundaries, then the grid-to-grid interpolations down the multigrid hierarchy are achieved with only simple line interpolations, as Fig. 1 shows. Moreover, by carrying auxiliary points in the form of L-shaped quartets just outside each side of each subdomain corner, we avoid the need for any further halo exchanges as this procedure proceeds down the hierarchy of scales (or, in its adjoint version, up the hierarchy).

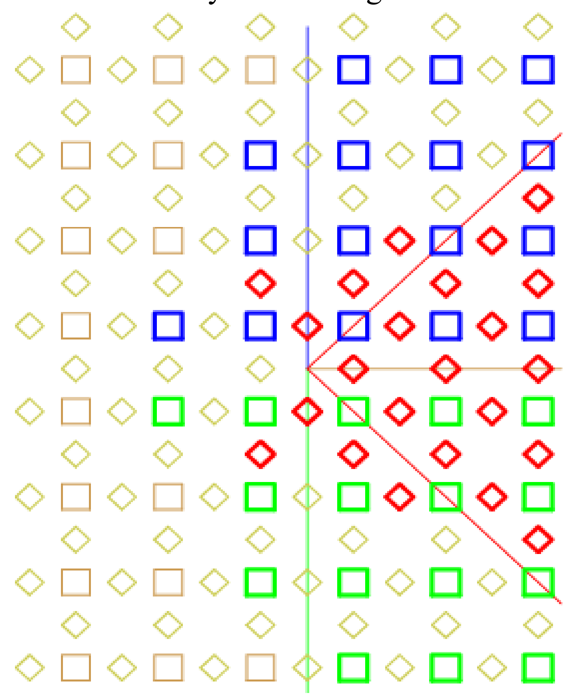

Fig.1: Arrangement near the corner of subdomains showing how interpolation from the coarser offset grid to the finer one, diagonally oriented to it, can be accomplished by 4-point interpolation along lines. When the L-shaped quartets of auxiliary grid values on both sides of each subdomain corner are appended, it is possible to carry out these interpolations without recourse to further lateral exchanges of data at each step in the hierarchy of grid-to-grid interpolations.

3. Estimating analysis error

We propose to approach the difficult problem of estimating the analysis error by employing the beta filter itself to first smooth the precision-weighted distribution of unit impulses at each observation location to obtain a precision-weighted data density. The filter in this case can be a single quasi-Gaussian at a relatively coarse scale taken from the multigrid formulation. Then, since at each grid point, we know the superposition weights used to characterize the background error covariance, and knowing the quasi-Gaussian form of the filters they are applied to, it is possible to perform a local virtual "analysis" in the Fourier domain that assumes homogeneity of both background covariance, and observation density and to extract a corresponding local spectral representation of the analysis error's spectrum – in effect, a power spectrum of the expected error in the local analysis. This can be projected onto the given space of the local multigrid scale weights to provide a fairly informative characterization of the spatial form of the expected analysis error, or we can simply extract the variance part of this calculation at each point. In this way, we should be able to use the machinery of the multigrid beta filter itself to extract from the given distribution of the data, and its known errors, a fair characterization of the optimal analysis error, which would otherwise be difficult to estimate.

References

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