Determining the cause-effect relationship between climatic variables based on time series: is it possible or not?

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According to paleoreconstructions and observational data for the late XX – early XXI centuries, it is noted that time series for global temperature *T* are generally leading the corresponding time series for the CO_2 content in the atmosphere *q* (references to the relevant works are given in [1, 2]). On this basis, it is sometimes stated that the modern theory of global warming, according to which the main reason for the ongoing increase in global temperature is CO_2 emissions into the atmosphere, is incorrect. These statements are made without involving any ideas about the structure of the Earth's system, but only on the basis that the "effect" (temperature) cannot lead the "cause" (CO_2).

The lag of *q* changes relative to *T* changes is also obtained in simulations with modern climate models (in particular, with the MPI-ESM general circulation model included in the CMIP-6 model comparison project, see fig. 1), in which the CO_2 greenhouse effect is the main cause of the ongoing global warming.

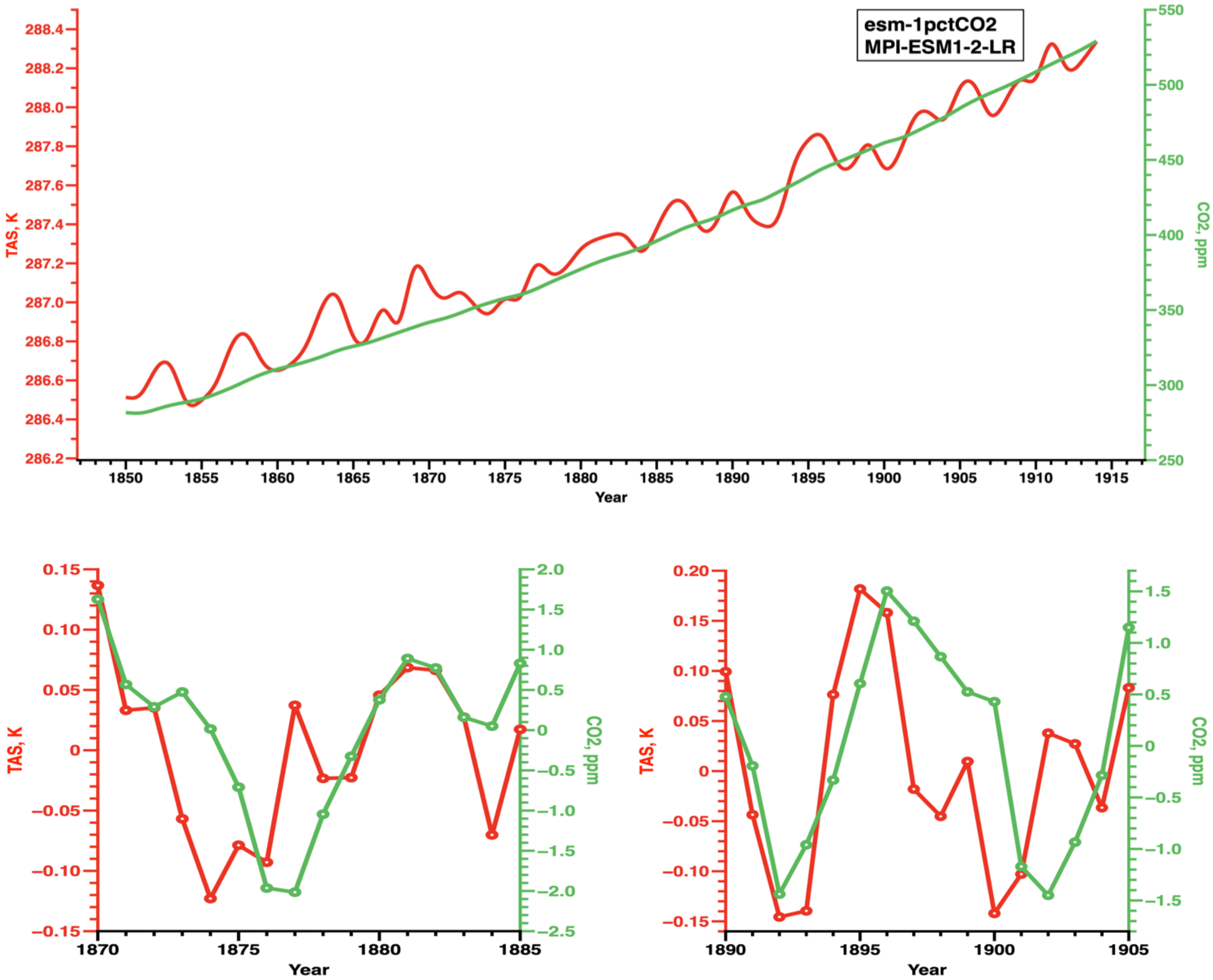

Fig. 1. Changes in global temperature (red) and CO_2 content in the atmosphere (green) in simulations with the MPI-ESM climate model. The upper part of the figure shows the initial series, the lower part shows the series with the linear trend removed.

This result illustrates the fact that time lag between changes in variables cannot be used to judge the nature of the causal relationship between them. Moreover, not only the time lag between the series, but also any other characteristics of these series do not allow us to draw a final conclusion about the nature of the causal relationship between the corresponding variables, without any ideas about the nature of the interaction of these variables or, to put it more formally, about the structure of the system in which they are exist. Let's show it.

Consider two arbitrary data series from a finite number of *n* elements:

$$X = [\ x_1,\ \dots,\ x_n\ ], \qquad (1)$$

$$Y = [\ y_1,\ \dots,\ y_n\ ]. \qquad (2)$$

If the values of all their elements are limited (this is always valid for real data series) and are non-zero (this can be achieved by choosing measurement units), then it is possible to build a system where changes in *X* are the cause of changes in *Y*, that is, changes in *X* affect changes in *Y*, and changes in *Y* do not affect changes in *X*. In the simplest case, this can be written as a differential equation:

$$\frac{dY}{dt} = f(X,t). \qquad (3)$$

Here *f* is some arbitrary function, *t* is time. Let function *f* be linear:

$$f(X,t) = K(t)X(t). \qquad (4)$$

Taking into account (3) and (4), we obtain

$$K(t) = \frac{Y'(t)}{X(t)}. \qquad (5)$$

Then, using (1) and (2), it is possible to obtain a number of specific values of *K* corresponding to the original series *X* and *Y*:

$$K_n = (y_{n+1} - y_n) / (x_n \Delta t). \qquad (6)$$

Here Δt is a time step. Thus, the system follows

$$y_{n+1} = K_n x_n \Delta t + y_n. \qquad (7)$$

Within this system, changes in *X* are the cause of changes in *Y*. Since the series were taken arbitrarily, in a similar way it is possible for the same series to construct a system in which, on the contrary, changes in *Y* will cause changes in *X*.

Thus, for any pair of series *X*, *Y*, whose values are nonzero and limited, it is possible to construct both a system in which *Y* is the cause and *X* is the effect, and vice versa. This means that on the base of data series analysis only, it is impossible to draw an unambiguous conclusion about the nature of the causal relationship between their changes, without assuming a physical theory of their interaction.

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References:

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2. *Muryshev K.E., Eliseev A.V., Mokhov I.I., Timazhev A.V., Arzhanov M.M., Denisov S.N.* Influence of Nonlinear Processes on the Time Lag between Changes in the Global Temperature and the Carbon Dioxide Content in the Atmosphere. *Dokl. Earth Sci.*, 2021, V. 501, P. 949–954.