Fourier analysis of climate change in Russia

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The archive of observations of temperature f(k) at synoptic stations located near major cities of Russia is analyzed. Period: 2001-2020. Observations at the stations are made 8 times a day with a step of 3 hours. In addition, the minimum and maximum temperatures per day are analyzed. At those moments k, when the measurement result for one or another reason did not get into the archive (for example, the checking algorithm recognized this measurement as erroneous), such a measurement is labeled $A(k)=0$. Otherwise, we assume $A(k)=1$.

We estimate in each year, in addition to the average temperature, the first Fourier harmonic for seasonal and daily fluctuations. To do this, we introduce a basis of five "almost orthogonal" grid functions (if there are no gaps in the archive of measurements, these functions are L^2 -orthogonal):

$$u_1 = 1,\ u_2 = \sin\left(2\pi k/8\right),\ u_3 = \cos\left(2\pi k/8\right),$$
$$u_4 = \sin\left(2\pi k/8N\right),\ u_5 = \cos\left(2\pi k/8N\right),$$

where N is the number of days in a year. Let's determine the best approximation of the function from the measurement data $f(k)$: $f(k) \approx \sum_{j=1}^{5} z_j u_j(k)$, $z_j = const$. We define matrix Q with elements $q_{ij} = \dfrac{\sum_{k=1}^{8N} A(k) u_i(k) u_j(k)}{\sum_{k=1}^{8N} A(k)}$, $i, j = 1,\dots,5$, and the vector $\vec{Z}$ with components $z_i = \dfrac{\sum_{k=1}^{8N} A(k) u_i(k) f(k)}{\sum_{k=1}^{8N} \mathrm{A}(k)}$.

The best coefficients $\{b_j\}_{j=1}^{5}$ are obtained by solution of the SLAE $Q\vec{b} = \vec{Z}$. The coefficients providing a minimum of the mean squared deviation $\sigma^2 = \sum_{k=1}^{8N}\left[f(k) - \sum_{j=1}^{5} b_j u_j(k)\right]^2$ are evaluated for any year and for any station. We obtain 5x20 values $\{b_j(m)\}_{j=1}^{5}$, $m = 1,\dots,M$, since the archive includes data for M=20 years.

Let us make a transformation for the coefficients b_4, b_5 to evaluate the amplitude of the season temperature dynamics $\Theta(m) = \sqrt{b_4^2(m) + b_5^2(m)}$ and its phase $\Phi(\mathrm{m}) = \arctan[b_5(\mathrm{m}) / b_4(\mathrm{m})]$. Similarly, we define the amplitude and phase for the day oscillations: $\vartheta(m) = \sqrt{b_2^2(m) + b_3^2(m)}$ and $\varphi(\mathrm{m}) = \arctan[b_3(\mathrm{m}) / b_2(\mathrm{m})]$.

Thus, we obtain 5 functions of the discrete time moment m: $\mathrm{g(m)}:\ b_1(\mathrm{m}), \vartheta(\mathrm{m}), \varphi(\mathrm{m}), \Theta(\mathrm{m}), \Phi(\mathrm{m})$. Then we construct for every function the best linear approximation: $g(\mathrm{m}) \approx g_0 + g_1\,\mathrm{m},\ g_0, g_1 = const$ by the formulae:

$$g_0 = \frac{1}{M}\sum_{m=1}^{M} g(\mathrm{m}),\quad g_1 = \frac{\sum_{m=1}^{M} g(\mathrm{m})\left[m - (M+1)/2\right]}{\sum_{m=1}^{M}\left[m-(M+1)/2\right]^2}.$$

Here, g_0 is equal to the mean value of the function $g(m)$ for M years, and g_1 is its tendency. The mean square deviation of the function is equal to: $\sigma_f = \sqrt{\dfrac{1}{20}\sum_{k=1}^{20}\left[g(k) - g_0 - g_1 k\right]^2}$. We define the trend as significant if the inequality for its significance of the estimation: $g_1\,\mathrm{M}/\sigma \ge 1$ is fulfilled.

Also we evaluated the minimal and maximal (for any day) temperature in a similar way, but without functions $u_2(k), u_3(k)$. Let $t_{fs}(\mathrm{m})$ be the day of the last spring frost ($t^{\circ}_{\min} < 0$) and $t_{fa}(\mathrm{m})$ be the day of the first autumn frost in the year m. We have also evaluated the trends of these dates over 20 years.

Results (see Table for some details). The average annual temperature (as well as the average annual $t^{\circ}_{\min}(\mathrm{m})$) grew in all cities, although not monotonously. The tendencies for $t^{\circ}_{\max}(\mathrm{m})$ are not so clear.

The amplitude of seasonal fluctuations decreased in most cities in the middle and high latitudes. In the southern cities (Astrakhan, Stavropol) the amplitude grew. In some cities, the amplitude trends were not significant. The mean daily amplitude significantly increased in Nizhny Novgorod, Stavropol and Astrakhan.

The time of the last spring and first autumn frosts is important for agriculture and transport. We learn that the trends of these dates have different signs. In most cities, this warm period between these frosts has increased. The most noticeable growth occurred in the northernmost city, Murmansk, where the increase is more than 1 month.

Conclusions. Fourier analysis of the series of temperature observations makes it possible to identify both coarse effects (an increase in the average air temperature) and more delicate ones (the amplitude and phase of daily and seasonal fluctuations). In the second case, the results are not so unambiguous and significantly depend on the location of the synoptical station. The same applies to the estimation of the dynamics of the minimal and maximal temperature per day, as well as the days of frost. Such climate changes are useful to take into account when making decisions in agriculture, utilities, transport, construction, medicine, etc.

The table shows the climatic data for megapolises. When analyzing other regions, the results may show different trends. For example, in the small settlement of Barabinsk (Siberia), no significant changes in the average annual temperature were found. More complete information about climate change is expected to be presented in another publication.

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Literature

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Table of tendencies (and the evaluation's significance)

City	Mean annual temperature (degree/year)	Amplitude of seasonal oscillations (degree/year)	Mean annual minimal temperature (degree/year)	Mean annual maximal temperature (degree/year)	Day of the last spring/ first autumn frost (day)	Increase of the warm periods (day)
Moscow	0.081 (3.3)	-0.066 (0.9)	0.085 (3.1)	-0.052 (0.7)	0.015/0.744	14.6
St. Petersburg	0.088 (3.0)	-0.100 (3.4)	0.098 (3.3)	-0.087 (1.2)	0.037/-0.349	-6.2
Penza	0.068 (2.4)	0.006 (0.1)	0.067 (2.1)	-0.009 (0.1)	0.07/0.067	1.2
Ekaterinburg	0.034 (1.0)	-0.002 (0.2)	0.034 (1.0)	0.015 (0.2)	-0.324/-0.123	6.0
Chita	0.056 (1.5)	-0.068 (1.4)	0.052 (1.5)	-0.057 (1.0)	0.133/0.077	-1.1
Khabarovsk	0.054 (1.8)	-0.048 (0.9)	0.157 (6.3)	-0.081 (1.5)	-0.001/0.282	5.6
Vladivostok	0.033 (1.2)	-0.014 (0.3)	0.107 (4.7)	-0.008 (0.2)	0.031/-0.145	-3.5
Magadan	0.063 (3.2)	-0.017 (0.6)	0.121 (5.1)	-0.013 (0.4)	-0.269/0.162	8.6
Stavropol	0.050 (1.7)	0.021 (0.4)	0.017 (0.7)	0.046 (0.8)	-0.03/-0.198	-4.0
Astrakhan	0.050 (2.0)	0.045 (0.8)	0.041 (1.6)	0.044 (0.8)	-0.045/-0.143	-2.0
N.Novgorod	0.053 (2.2)	-0.06 (0.8)	0.042 (1.8)	-0.045 (0.6)	0.074/0.549	9.5
Murmansk	0.054 (1.5)	-0.014 (0.3)	0.05 (1.5)	0.007 (0.1)	-0.824/0.736	31.2