Localization of satellite data

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When processing satellite information about various geophysical fields, it may be necessary to convert the fields of values averaged by pixels by satellite device into fields of local values on the output geographical grid. Sometimes it is necessary to inverse the transformation of the local values on the grid into a set of pixel integrals. Compact finite-difference schemes provide a higher accuracy order than classical explicit algorithms. In order to convert one type of field into another, it is sufficient to solve a system of linear algebraic equations (SLAE) with a sparse matrix. The transformations are not exact for an arbitrary field, and lead to errors that depend on the grid step and the wave spectrum of the interpolated fields. These errors for different algorithms are compared using the Fourier analysis. The comparison confirms the advantage of compact algorithms for a wide spectral range of waves. The advantage of the compact algorithms is also observed for Gaussian formulas, but in some special way. The grid knots for such formulae are not equidistantly spaced. Additional difficulties in converting one type of field to another exist in the vicinity of the boundary of the computational domain V. Modification of these algorithms is necessary here. For many geophysical problems, the spectral energy distribution of the interpolated fields is known *a priori* if these fields are interpreted as random. It is useful to apply this a priori information for interpolation to minimize the probabilistic error for a given type of field. Compact finite-difference schemes for these problems also provide minimal error.

Let's divide the area V into small squares (pixels) with a side h. Let N be the number of pixels at each coordinate. We use as input data the pixel-averaged values of a function $u(x,y)$ (e.g. temperature) $\left\{I_{j,k}\right\}_{j,k=1}^{N}$, where $I_{j,k}[u]=\int\limits_{(j-1/2)h}^{(j+1/2)h}\int\limits_{(k-1/2)h}^{(k+1/2)h} u(x,y)dxdy$ and evaluate the function u values $\left\{u_{jk}\right\}_{jk=0}^{n}$ in the grid's knots $\langle jh,kh\rangle$ (with the step h) that are in the pixels' centers. Let us determine by 2D Fourier transform $F_{y\to\eta}F_{x\to\omega}$ the symbol of the reference functional (of the integral by square pixel). Then we obtain:

$$\sigma_{reference}(\omega,\eta)=\frac{\sin(\omega h/2)\sin(\eta h/2)}{(\omega h/2)(\eta h/2)}. \quad (1)$$

We consider for the evaluation the nine-point stencils for both fields for creating a compact scheme. Let's shift the origin to the center of the square with coordinates <*j,k*> and use symmetry, with respect to the coordinate axes, and with respect to the diagonals:

$$aI_{0,0}+ b[I_{-1,0}+I_{0,-1}+I_{1,0}+I_{0,1}]/4+c[I_{-1,-1}+I_{1,-1}+I_{-1,1}+I_{1,1}]/4$$
$$=u(0,0)+p\left[u(0,-h)+u(0,h)+u(-h,0)+u(h,0)\right]+$$
$$+q\left[u(-h,-h)+u(-h,h)+u(h,-h)+u(h,h)\right]/4. \quad (2)$$

Let us determine 5 coefficients: *a, b, c, p, q*. We will assume that Eq. (2) is fulfilled on the monomial functions: $u=1, x^2, x^4, x^2y^2$ etc. To ensure the 4-th order of accuracy (in reality, the 5th due to the symmetry of scheme (2)), we need to use these 4 test functions, and to improve the order up to the 6-th we need to add x^6, x^4y^2. However, 5 coefficients in (2) are not sufficient for the goal, and we must limit ourselves to the 4th order. We obtain by substitution into compact relation (2) the following SLAE:

$$1\Rightarrow h^2(a+b+c)=1+p+q, \quad (3)$$
$$x^2\Rightarrow h^4(a+7b+13c)/12=h^2(2p+4q)/4, \quad x^4\Rightarrow$$
$$h^6\left[a+b(2+2\cdot121)/4+c(4\cdot121/4)\right]/80=h^4(2p+4q)/4,$$
$$x^2y^2\Rightarrow h^6\left[a+b(4\cdot13)/4+c(4\cdot169)/4\right]/144=h^4 4q/4.$$

To determine the local values of the function $u(x,y)$ by their mean values we need “global” SLAE (2). Its order is equal to the number of the grid knots. We need to modify equations (2) near the boundary of V.

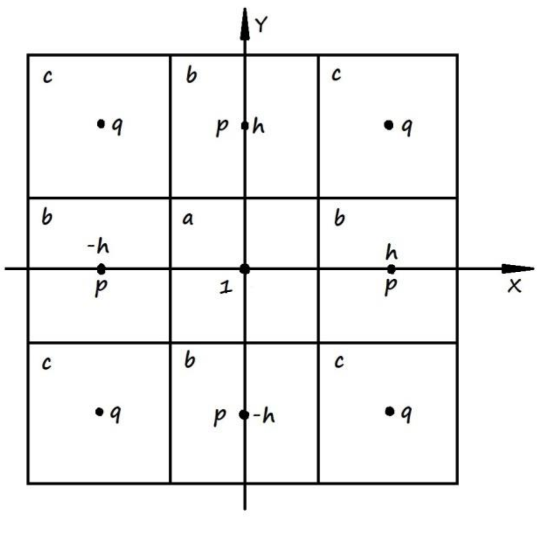

Fig.1. The weights for integrals by pixels and the coefficients for the values in knots in relation (2)

In each row of the matrix there is one element equal to 1, 4 elements with the value p and 4 with the value q. The remaining elements of the row are zero. If

p or q is also zero, then the matrix is sparser and it will be cheaper to solve SLAE (2). We obtain from "local" SLAE (3) the coefficients for "global" SLAE (2) for the normalization q=0:

$$a=\frac{247}{198h^2},\ b=\frac{31}{198h^2},\ c=\frac{-1}{72h^2},\ p=\frac{9}{44}.$$

The inequality $4|p|<1$ is fulfilled, and therefore (according to the Gershgorin theorem) the principal diagonal of the matrix dominates and the matrix of SLAE (2) is non-degenerate (if the boundary condition for the problem is suitable).

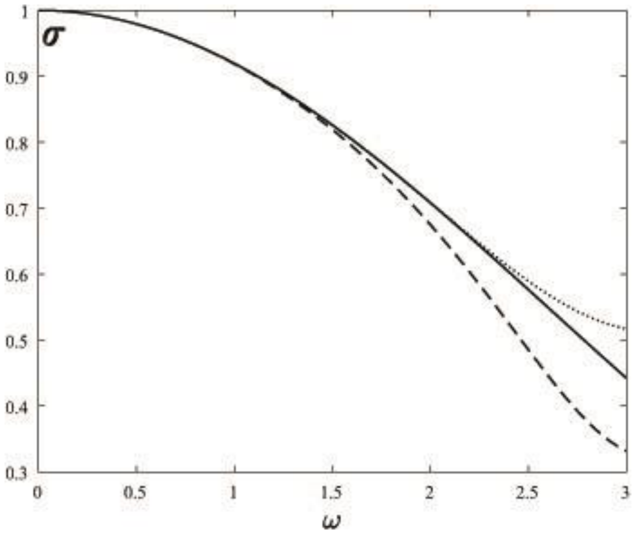

Fig.2. Solid line - symbol of the reference functional; dotted line - compact approximation for c=0; dashed line – for q=0. All the symbols are shown on the diagonal, i.e. when $\omega=\eta$.

Vice versa, if we determine the set of the integrals $\{I_{j,k}\}$, by the local values of the function, the normalization c=0 is preferable. We obtain the solution of SLAE (3):

$$a=\frac{258}{209h^2},\ b=\frac{102}{209h^2},\ p=\frac{140}{209},\ q=\frac{11}{209}.$$

The principal diagonal is dominant again, since $|a|>|b|$ here. The matrix of SLAE (2) is non-degenerate (if the boundary condition for the problem is suitable). The plots in Fig.2 confirm the high quality of the compact approximation for a wide wave range. The naive approximations (the integral is proportional to the average value of the function in the corners of the square): $I_{00}\approx h^2[u(-h/2,-h/2)+$

$$u(-h/2,h/2)+u(h/2,-h/2)+u(h/2,h/2)]/4 \quad (4)$$

and $u(0,0)=\alpha I_{0,0}+\beta\left[I_{0,-h}+I_{0,h}+I_{-h,0}+I_{h,0}\right]$, (5)

where $\alpha=5/\left(3h^2\right),\ \beta=-1/\left(6h^2\right)$. The corresponding symbols are shown in Fig.3, where the differences with the reference solution are much greater than in Fig.2.

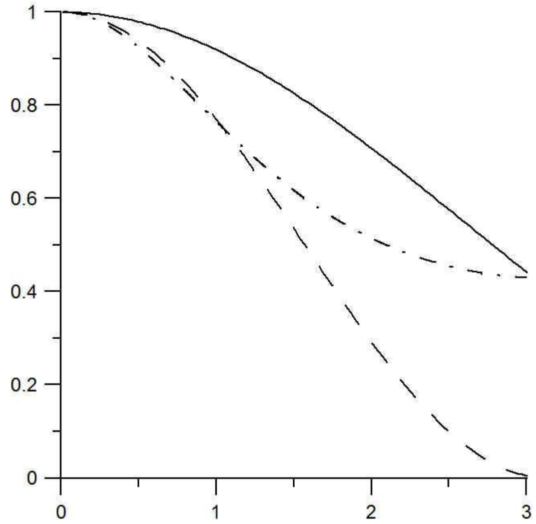

Fig.3. Symbols for the naive formulae. Solid line – ibid, dash dotted line – for formula (4), dashed line – for formula (5).

Thus, the compact formulae for the problems are much more accurate. The additional computation (the sparse matrix inversion) is not expensive.

Literature

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