Comparisons of H_2O pathways with moist isentropes.

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1) Motivations - Introduction.

A moist isentropic framework is used in Bailey *et al.* (2019, hereafter B19) to study poleward moisture transport in the atmosphere. In this work, it was demonstrated that the moisture plumes evaporated from 10° bands of latitude from the subtropics poleward approximately align with the "moist-air isentropic surfaces," as measured by the first-order approximation of the equivalent potential temperature:

$$\theta_e \approx \theta \, \exp\left(\frac{L_v \, r_v}{c_{pd} \, T}\right) \approx \theta \left(1 + \frac{L_v \, r_v}{c_{pd} \, T}\right) \approx \theta + \frac{L_v \, r_v}{c_{pd}} \, ,$$

where T is the absolute temperature, p is the pressure, $p_0 = 1000$ hPa, $\theta = T \, (p_0/p)^\kappa$ is the dry-air potential temperature with $\kappa \approx 0.285$, $c_{pd} \approx 1006$ J K^{-1} kg^{-1} is the dry-air specific heat at constant pressure, $L_v \approx 2500$ kJ kg^{-1} is the latent heat of vaporization and r_v is the water-vapour mixing ratio.

However, as demonstrated for the 30°S to 40°S latitude band in Fig. 1 (Fig.1e of B19), the colored moisture plume exhibits a southward shift from $\theta_e \approx 310$ K below 850 hPa to $\theta_e \approx 300$ K above 500 hPa, indicating some degree of "cross-isentropic" transport which may be interpreted as a "diabatic" decrease of θ_e through water loss through precipitation or radiative cooling (OLR).

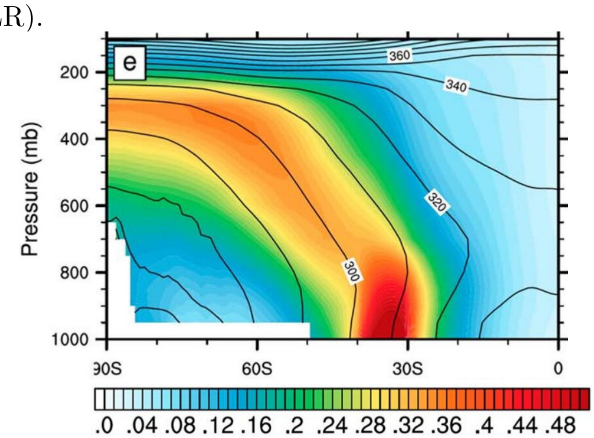

Figure 1: *The Fig.1e of B19. Normalized, annual and zonal mean distributions of water vapor in the Southern Hemisphere sourced from 30°S to 40°S, overlain with equivalent potential temperature θ_e contours (K). All variables come from a fully coupled Community Earth System Model.*

It is shown in this note that the way the isentropes are measured matters in the interpretation of "cross-isentropic" transport and that, in fact, this "cross-isentropic" transport almost vanishes if the absolute definition of moist-air entropy is used instead of equivalent potential temperature.

2) The absolute moist-air entropy.

Absolute definitions of reference entropies of dry air and water vapour have been considered in atmospheric science by Hauf and Höller (1987), Marquet (2011) and Stevens and Siebesma (2020), with a recent application in the IFS model described in Marquet and Bechtold (2020). The first-order approximation of the corresponding potential temperature θ_s is computed in Marquet (2011):

$$\theta_s \;\approx\; \theta \, \exp\left(- \, \frac{L_v \, q_l}{c_{pd} \, T} \; - \; \frac{L_s \, q_i}{c_{pd} \, T} \; + \; \Lambda \, q_t\right) \, , \qquad (1)$$

where $\kappa \approx 0.2857$, $c_{pd} \approx 1004.7$ J K^{-1} kg^{-1}, q_l, q_i and $q_t = q_v + q_l + q_i$ are the liquid-water, ice and total water specific contents, respectively, $L_s \approx 2835$ kJ kg^{-1} is the latent heat of sublimation and $\Lambda = (s_{vr} - s_{dr})/c_{pd} \approx 5.87$ is the key value depending on the absolute definitions of the reference entropies $s_{vr} \approx 12673$ J K^{-1} kg^{-1} and $s_{dr} \approx 6777$ J K^{-1} kg^{-1} .

3) Comparison of moist-air isentropes

Fig. 2 shows the same moisture plume and contours of θ_e (solid lines) as in Fig. 1, but with the new contours of θ_s added (dashed lines). Since the moisture plume almost follows the $\theta_s = 300$ K contour, the "cross-isentropic" transport almost vanishes with the absolute-entropy potential temperature θ_s.

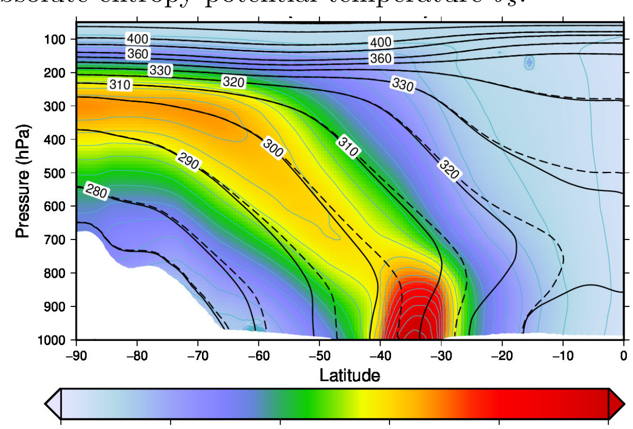

Figure 2: *The moisture plume evaporating from the latitude band 30° S to 40° S with the isentropes computed using either θ_e (solid lines) or with the new absolute moist-air entropy potential temperature θ_s (dashed lines).*

The isentropic means of the moisture plume of Fig. 2 are depicted in Figs. 3 as a function of height. They are computed according to Eq.(1) of Pauluis and Mrowiec (2013) for each bin of 1 K for both θ_e (top) and θ_s (bottom), with computations adapted here to the latitude and pressure coordinates.

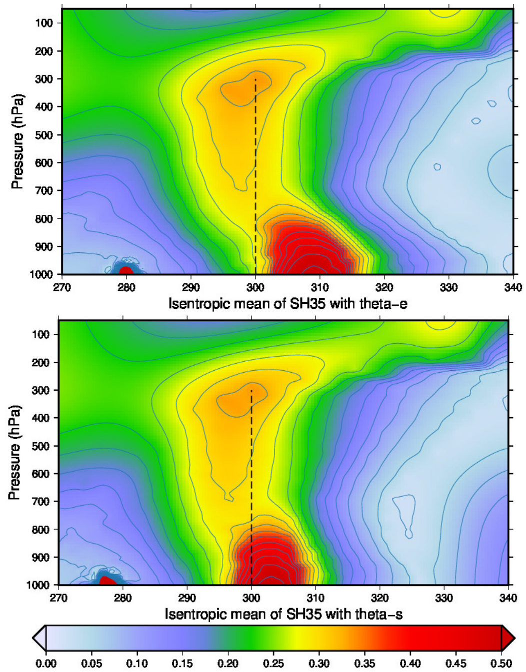

Figure 3: *"Isentropic means" of the moisture plume of Fig. 2 averaged over "isentropic surfaces" for θ_e (top) and θ_s (bottom).*

Clearly, the impact of the diabatic heating rate $\dot{Q}$ defined from the absolute entropy equation $d\theta_s/dt = \dot{Q}/T$ and due to radiation, evaporation, precipitations and irreversible changes of phases, is smaller for θ_s than for the alternative definitions based on the "equivalent" values $d\theta_e/dt$ and θ_e

This feature can be better understood with the definition $s(\theta_s) = c_{pd} \ln(\theta_s) + s_{ref}$ of Marquet (2011, Eq. 59), where θ_s is directly linked to the absolute moist-air entropy $s(\theta_s)$ by a simple relation where $s_{ref} \approx 1139$ J K^{-1} kg^{-1} is a constant and c_{pd} is the same constant as in Eq. (1). The link $s(\theta_e) \approx c_{pm}(q_t) \ln(\theta_e/T_0)$ between θ_e and $s(\theta_e)$ is more complicated (Eq. 13 of Marquet, 2017, with $T_0 = 273.15$ K), because it involves additional term depending on changes on q_t in the moist-air value $c_{pm} = c_{pd} + (c_l - c_{pd})\, q_t$, where $c_l \approx 4218$ J K^{-1} kg^{-1} is the liquid-water value. According to Eq. 48 of Marquet and Geleyn (2015) and Eq. 6 of Marquet and Dauhut (2018), changes in θ_s and θ_e are given by

$$\frac{c_{pd}}{\theta_s}\,\frac{d\theta_s}{dt} = \frac{ds}{dt} = \frac{\dot{Q}}{T}\,, \tag{2}$$

$$s(\theta_e) = s(\theta_s) + (s_{dr} - s_{lr})\, q_t - s_{dr}\,, \tag{3}$$

$$\frac{c_{pm}}{\theta_e}\,\frac{d\theta_e}{dt} = \frac{\dot{Q}}{T} + \left[\, s_{dr} - s_{lr} - (c_l - c_{pd})\,\frac{s(\theta_e)}{c_{pm}} \right] \frac{dq_t}{dt}\,, \tag{4}$$

where $s_{lr} \approx 3517$ J K^{-1} kg^{-1} is the reference absolute entropy of liquid water.

For varying total water content and $dq_t/dt \neq 0$ like in the moisture plumes in Figs. 2 and 3, the additional bracketed terms in Eq. (4) prevent a clear interpretation of $d\theta_e/dt$, as compared to the sole diabatic term $\dot{Q}/T$ in Eq. (2). The impact of the positive bracketed term should explain why the moisture plume does not follow contours of θ_e, if the physical processes almost lead to a conservation of θ_s. More precisely, for $d\theta_s/dt \approx 0$ and thus $\dot{Q} \approx 0$, if q_t decreases with height then $dq_t/dt < 0$ and $d\theta_e/dt < 0$, with indeed a decrease in θ_e by about 10 K from the surface to 400 hPa in the figures.

4) Conclusions.

The results of this note show that the way in which the moist-air isentropes are defined and calculated has a strong influence on the interpretation of cross-isentropic and diabatic processes in the study of poleward atmospheric moisture transport.

The absolute definition of entropy and the use of the associated variable θ_s, instead θ_e, leads to original, simpler and more accurate physical interpretations.

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