Effect of supersaturation constraint in a variational data assimilation system

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1 Introduction

The reproducibility of precipitation in the early stages of forecasts (spin-down or spin-up problem) has been a significant issue in numerical weather prediction. This issue is thought to be caused by moisture imbalance in the initial data given by data assimilation. In the case of the Japan Meteorological Agency (JMA) mesoscale data assimilation system JNoVA, we found that the imbalance stems from the existence of unrealistic supersaturated states in the minimal solution of the cost function in JNoVA. We implemented a penalty function method for the mixing ratio within JNoVA to suppress unrealistic supersaturated states and investigated the effects on the reproducibility of precipitation.

2 Method

The fundamental solution for handling the moisture imbalance problem are to construct proper control variables and set proper background error covariance (see, [1]). However, they are quite difficult, thus we apply an exterior penalty function method to avoid generating unrealistic moisture states and to obtain appropriate moisture balance, without entering into the problem of reconfiguring the control variables and error covariances.

An exterior penalty function method is one of the numerous algorithms to solve a constrained optimization problem [3, 2]:

$$\min_{\mathbf{x}\in X} f(\mathbf{x}) \quad \text{subject to } g(\mathbf{x}) \le 0, \tag{1}$$

where $f(\mathbf{x})$ is the objective function (or cost function), $g(\mathbf{x})$ is the (nonlinear) constraint function, and X is the control space. In this method, the original constrained problem (1) is converted to an unconstrained problem

$$\min_{\mathbf{x}\in X} f_2(\mathbf{x}), \tag{2}$$

by introducing the auxiliary function defined by

$$f_2(\mathbf{x}) = f(\mathbf{x}) + \lambda \max\{0, g(\mathbf{x})\}^{\alpha}, \tag{3}$$

where $\lambda > 0$ and $\alpha \ge 1$ are the penalty parameters. The second term of equation (3) is called the "penalty term" or "penalty function". If there is more than one constraint, one additional penalty term is added for each constraint.

The implementation of the exterior penalty function method in the variational assimilation system is very simple, it is done by just adding penalty term

$$J_{\mathrm{qv}}(\mathbf{x}) = \lambda \sum_i \left(\max\{0, g_{1i}(\mathbf{x}), g_{2i}(\mathbf{x})\}\right)^{\alpha} \tag{4}$$

to the original cost function, where

$$\begin{cases} g_{1i}(\mathbf{x}) & = \mathrm{qv}_i(\mathbf{x}) - \mathrm{qvs}_i(\mathbf{x}) \\ g_{2i}(\mathbf{x}) & = -\mathrm{qv}_i(\mathbf{x}). \end{cases} \tag{5}$$

Here, qv_i and qvs_i are the water vapor mixing ratio and the saturation mixing ratio, respectively, at the i-th grid point. Here, $g_{1i} \le 0$ is the constraint for supersaturation at the i-th grid point, and $g_{2i} \le 0$ is that for the negative mixing ratio. Since the mixing ratio has large values in the lower troposphere, this construction of the penalty function (5) is intended for the effective modification of the atmospheric fields in the lower troposphere, which is closely related to the initiation and development of deep convection and the generation of precipitation.

3 Results and Summary

To investigate the impact of the penalty function method on the analysis and forecast, we conducted twin data assimilation cycle experiments from June 28th to July 8th, 2018. In the following, the experiment that employs the original JNoVA system is called "Ctrl," and those utilizing the new JNoVA system equipped with the penalty term J_{qv} are called "Tests." In the Tests, we set $\alpha = 1$ and $\lambda = 100, 200, 500$, and 1000, which are labeled "L100," "L200," and so on.

Figure 1 shows the impact of the penalty function on the modification for the violation defined by $\max\{0, g_{1i}(\mathbf{x}), g_{2i}(\mathbf{x})\}$. The violations are substantially reduced by the penalty function method as the value λ becomes large.

Figure 2 shows the three-hour accumulated precipitation in the forecasts from the initial data at 09 UTC on June 28th, which is the result of the first cycle. Strong rainfall along the baiu front in the sea northwest of Kyushu, in the radar/raingauge analyzed precipitation data (RA, treated as the observations), is not adequately reproduced in the forecast of Ctrl. In the forecast of the Tests, the reproducibility (including the location, distribution, and amount) of precipitation is improved. The distributions of precipitation in the Tests (L200, L500, and L1000) are similar to each other, and the maximum amounts of the precipitation in L500 and L1000 are almost comparable to that of the RA.

We also performed verifications of the 12-hour precipitation forecasts of Ctrl and the Tests every 12 hours, during the cycle period (the initial times are 00UTC and 12UTC). The fractions skill scores (FSSs) with a 10 km verification grid and thresholds of 1.0 mm/h and 10.0 mm/h are shown in Figure 3. We can see clear improvements in the FSSs of the Tests at both thresholds in the early stages of the forecast. For other cases with different verification grid sizes and thresholds, we confirmed that the Tests are superior to Ctrl in general (not shown). But it is difficult to determine what value of λ gives the best improvement in the FSS, since the scores vary with the threshold and the atmospheric state even though L1000 seems to be better among the Tests from Fig. 3. These indicate that the improvement is robust for the values of λ. One possibility is that the value $\lambda = 100$ is sufficient in the case of cycle assimilation because the differences of the atmospheric fields among the Tests are small compared to the differences between Ctrl and the Tests (not shown).

From these results, we conclude that the new moisture balance introduced by the penalty function method has a positive impact on the reproducibility of precipitation in the early stages of forecasts.

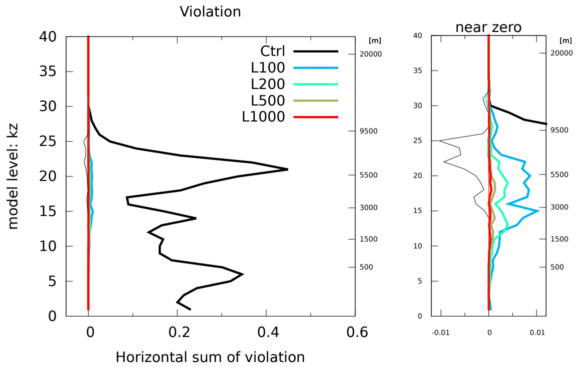

Figure 1: Vertical profile of horizontal summation of the violations with an enlarged view shown in the panel to the right.

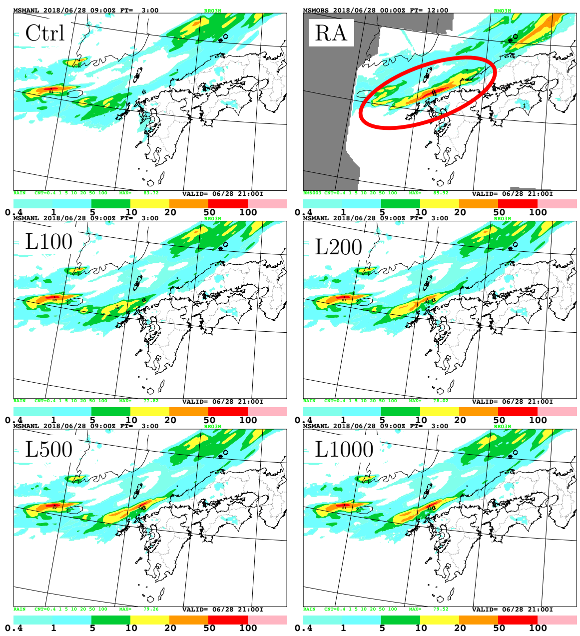

Figure 2: Three-hour accumulated precipitation ([mm/3h]; shaded as in the color bar) at 12UTC on June 28th, 2018. The different panels show the radar/raingauge analyzed precipitation (RA) and the forecasts of Ctrl and the Tests (L100, L200, L500, and L1000).

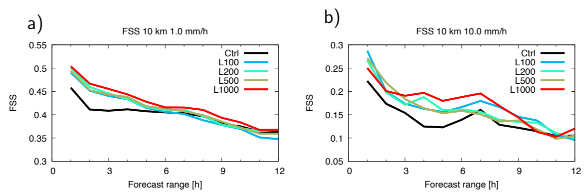

Figure 3: Fractions Skill Scores (FSSs) of the forecasts up to 12 hours. The verification grid is 10 km, and the precipitation thresholds are a) 1.0 [mm/h] and b) 10.0 [mm/h].

References

[1] R. N. Bannister, H. G. Chipilski, and O. Martinez - Alvarado. Techniques and challenges in the assimilation of atmospheric water observations for numerical weather prediction towards convective scales. *Quart. J. Roy. Meteor. Soc.*, 146:1–48, 2019.

[2] M. S. Bazaraa, H. D. Sherali, and C. M. Shetty. *Nonlinear programming: Theory and Algorithms.* John Wiley and Sons, 3 edition, 2006.

[3] D. P. Bertsekas. *Constrained Optimization and Lagrange Multiplier Method.* Academic Press, 1982.