Multigrid Beta function approach for modeling of background error covariances in the Real Time Mesoscale Analysis

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The Real Time Mesoscale Analysis (RTMA) and UnRestricted Mesoscale Analysis (URMA) projects provide hourly and sub-hourly analyses of surface variables, cloud cover and precipitation on the National Weather Service's National Digital Forecast Database grid. They are used for situational awareness, current conditions for transportation customers, verification of forecasts and as the reference for the bias correction in the National Blend of Models.

One major development underway is a fully three-dimensional "3D RTMA" system, which should replace the current two-dimensional (2D) system within the next year or two. It will provide 3D analyses at very high horizontal resolutions (~1.25 km) issued at very frequent time intervals (~15 min). One key prerequisite for the success of such an enterprise is a vastly improved efficiency in producing those analyses. The new approach to modeling of background error covariances, which will be discussed here, is one of the key components for the success of that effort.

Recursive filters (Purser et al. 2003a, b) represent efficient and very good approximations to the Gaussian components of which each univariate covariance operator is composed, and they accommodate spatial inhomogeneity and local anisotropy of analyzed field increments. On the down side, a recursive filter, being inherently sequential and with infinite support, is very difficult to successfully parallelize. Also, it is not easily able to describe covariances across various scales, or to take into account cross-correlations between different variables, or to provide the negative side-lobes which realistic covariance often possess.

Our alternative to recursive filters is based on the Beta distribution filters. The "radial" version of the Beta filter is defined, in any number of dimensions, as having the smoothing kernel:

$$\beta(\boldsymbol{r}) = \begin{cases} (1-\rho)^p, & \rho \le 1 \\ 0, & \rho > 1 \end{cases} ,$$

where p is a small positive integer exponent and, in the isotropic case,

$$\rho = \frac{1}{s^2}\boldsymbol{r}\cdot\boldsymbol{r}^T .$$

Here, s is a radial scale and $\boldsymbol{r}$ the displacement vector, e.g., in 2D, $\boldsymbol{r}=(x,y)$. Such a β function also has a quasi-Gaussian shape, but with a finite support. In an anisotropic generalization s^2 is replaced by a symmetric, positive-definite "aspect tensor", used as its matrix inverse $\mathbf{A}^{-1}$, so that:

$$\rho = \boldsymbol{r}\mathbf{A}^{-1}\boldsymbol{r}^T .$$

A larger exponent p implies a more Gaussian shape, but also a narrower one. The Beta filter is further used at a hierarchy of different scales, suitably weighted and combined in a multigrid scheme, in order to achieve a larger coverage and potentially a more versatile synthesis of anisotropic covariances, and allowing a greater control over the shape. Each successive "generation" of the multi-grid hierarchy, is characterised by successive factors of two in grid spacing and scale going from fine to coarse. Each generation is given separate weights, which signify the effect of these scales in the final solution.

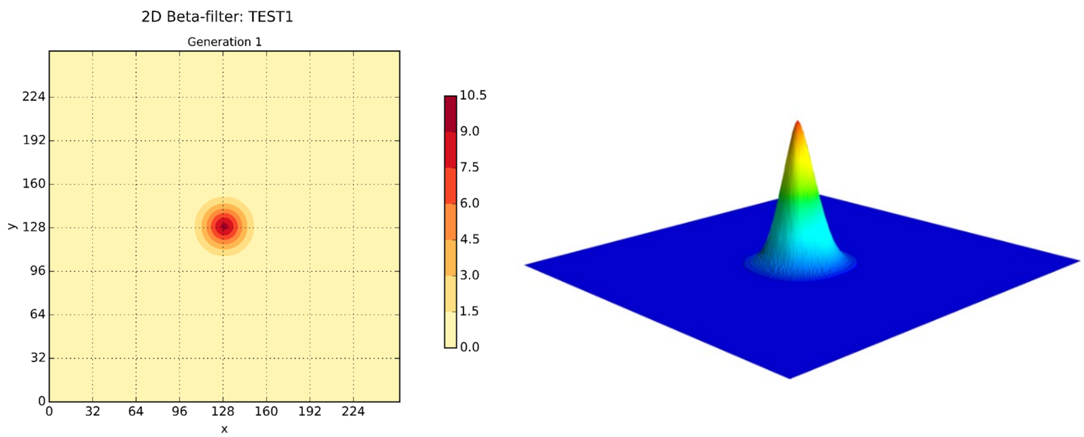

Fig. 1. Appearance of delta function impulse after application of the multigrid Beta filter.

A very efficient, "line" version of the filter is also being developed which, in small numbers of sequential combinations, can fill out the higher dimensions of a covariance operator (up to four dimensions), to eventually supersede the less efficient radial versions of the corresponding filter in final applications. Among other novelties, we are planning to introduce a new efficient method for normalization of covariances and a neural network approach for determining scale weights. The versatility of the approach will also allow as to introduce and experiment with the cross-covarianes in the definition of the background error covariance by replacing scalar weights acting on variables separately, by self-adjoint differential operators acting on several variables together. Other plans include application of this technique on the cubed-sphere, which will enable running data assimilation directly on that geometry, and an application within the new Joint Effort for Data assimilation Integration (JEDI) based on object-oriented principles.

References

Purser, R. J., W.-S. Wu, D. F. Parrish, and N. M. Roberts, 2003: Numerical aspects of the application of recursive filters to variational statistical analysis. Part I: Spatially homogeneous and isotropic Gaussian covariances. *Mon. Wea. Rev.,* **131,** 1524-1535.

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