A scale separation method for application of fourth-order advection with physics

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ABSTRACT

Although in idealized test studies fourth-order advection reduces the phase error of the solution, there are many reports that in NWP models with full physics it does not deliver the expected benefits. This study is a preliminary investigation of a method of scale separation where the "large scales" of motion are treated using the fourth-order advection scheme, while "short scales", those mostly affected by physics and orography influences, use the second-order scheme.

High-order, fourth- (e.g., Rančić 1988; Abramopoulos 1991; Janjic et al. 2011), and, in some instances, sixth-order (Chu and Fan 2001) approximations to horizontal advection, have been designed with the idea of providing a more accurate propagation of atmospheric systems and ocean flows in numerical simulations, by reducing the computational phase speed error. In idealized tests, high-order advection schemes indeed improve the solution, in accordance with their smaller truncation errors. However, though occasional reports confirm the benefits of high-order schemes in models with full physics (e.g., Juang and Hoke 1992; Skamarock and Klemp 2008), there is a general sense that they have not demonstrated the expected advantages.

A fourth-order scheme generates computational noise and its interference with the physics forcing, which takes place at the short end of spectrum, may explain to some extent the observed underperformance. Another factor, suggested by Janjic et al. (2011), is that the fourth-order scheme, due to a steeper inclination of the relative phase speed, has more profoundly wrong group velocity in the short portion of the wave spectrum than a corresponding second-order scheme. This means that a package of short waves, created by the model's physics, propagates more in the wrong direction in the case of the fourth- than in the case of the second-order scheme, which offsets the formally higher accuracy and negatively affects the overall performance.

One solution to this problem may be to introduce a nonlocal, "horizontally aware" physics, which was prophesized as inevitable for future high-resolution simulations by Arakawa (2000). Alternatively, one can spread the effect of physics forcing to the neighboring grid boxes through the application of spatial filters. In this short note a different solution is presented where the "large scales", which could be simply defined using a low-pass spatial filter of the considered field, are advected using the fourth-order scheme and the remaining field, the "perturbation", is advected using the second-order scheme.

An example of a one-dimensional version of the linear advection test suggested in Janjic et al. (2001), using, respectively, the second-, fourth- and combined schemes, is shown in Fig. 1 A short wave perturbation imposed on the triangle mimics the physics. The fourth-order scheme does not perform better than the second-order one. However, the combined scheme is able to provide a generally best solution. Preliminary testing has been done using the global NNMB-UJ model (Rančić et al. 2017). The objective, however, is to investigate the scale separation method as well as other mentioned approaches in order to successfully implement a high-order finite volume method (e.g., Ullrich et al.. 2010) in the new EMC's global FV3 model.

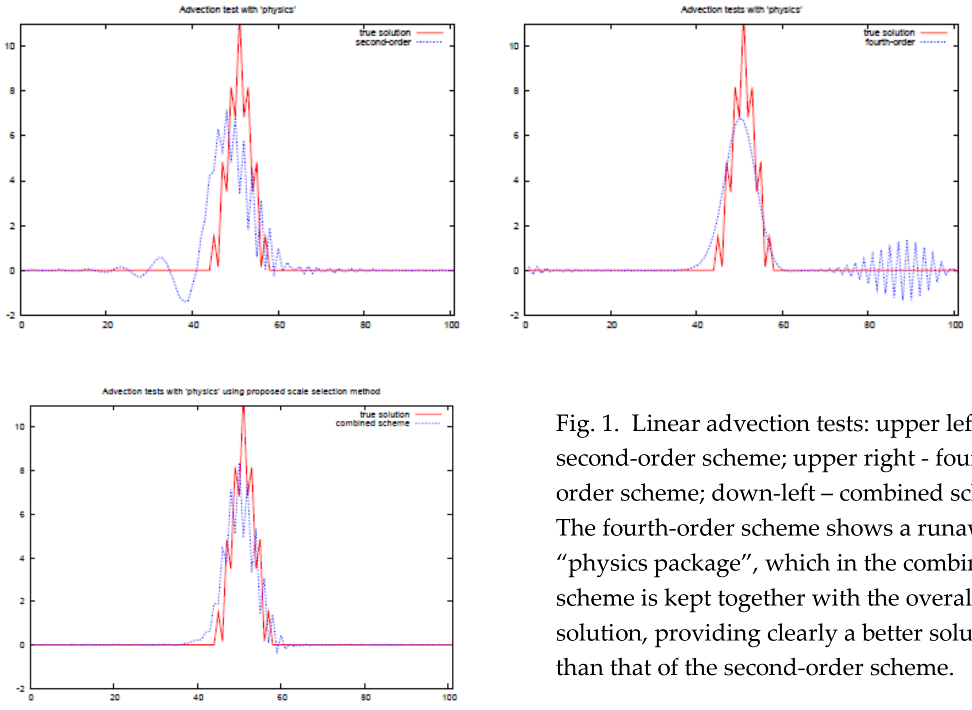

Fig. 1. Linear advection tests: upper left - second-order scheme; upper right - fourth-order scheme; down-left – combined scheme. The fourth-order scheme shows a runaway “physics package”, which in the combined scheme is kept together with the overall solution, providing clearly a better solution than that of the second-order scheme.

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