Conformal localized-overset polyhedral global grids based on Riemann surfaces

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1. Introduction

The age of massively-parallel computing brought about a resurgence of interest in using polyhedral grids, especially the cube and the icosahedron, as the computational horizontal frameworks of global atmospheric models (Ronchi et al., 1996; McGregor 1996; Rančić et al., 1996). Such grids are quasi-uniform and, unlike the latitude-longitude framework, do not suffer communications burden of operators (such as the Fourier filters) needing an immediate global reach.

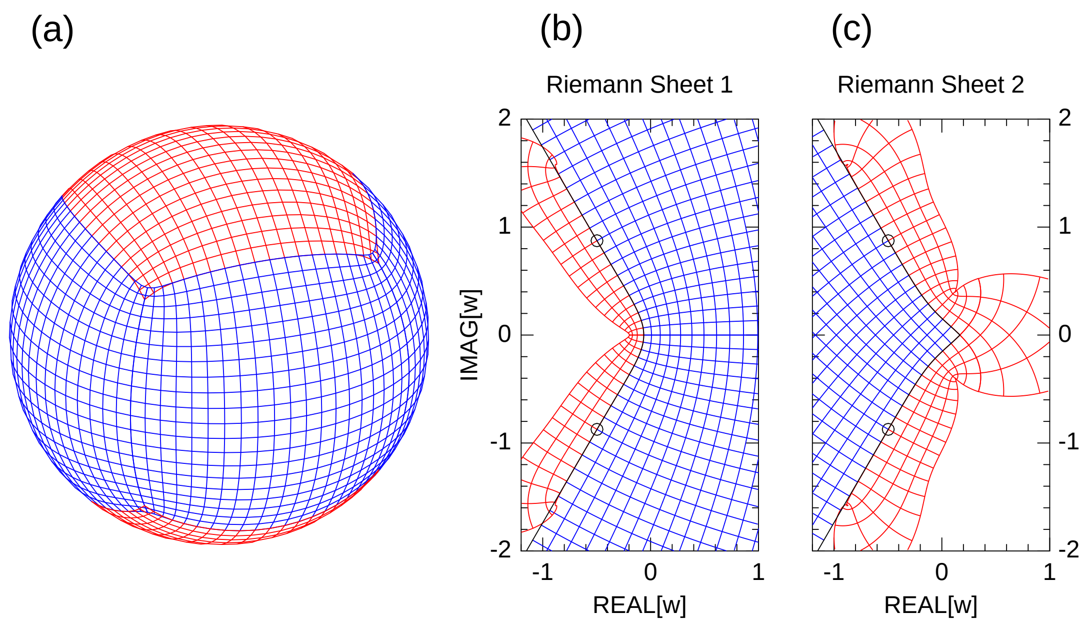

Figure 1. (a) A conformal cubed-sphere grid with Riemann surface corner oversets. (b) and (c) show two sheets of the associated idealized planar Riemann surface.

But a polyhedron has edges and corners, which poses other numerical difficulties. Rančić et al. (1996) and Purser and Rančić (1998) eliminated the edge discontinuities by adopting conformal mapping schemes, but intense grid curvature and unbounded resolution is a consequence of these mappings around the singularities that remain – the corners. This renders these otherwise attractive grid geometries disadvantageous in their original forms.

In order to achieve the considerable benefits of having exactly conformal grids, but without the problems associated with strong corner singularities, it is possible to "overset" separate conformal grids whose singularities are banished to distant regions of each tile, beyond the highly regular regions of active numerical interest where curvature and resolution excursions remain small. One way to do this is by using two conformal Mercator grid overset in the so-called "Yin-Yang" configuration (Kageyama and Sato 2004). But we still pay the price of having to carry out reconciling interpolations and blending around the entire globe-girdling zone of overlap.

An attractive variant suggested by the methods of complex analytic function theory by which conformal mappings are generated, is to exploit the feature of "Riemann surfaces" that are associated with mappings in which the domain of the function comprises two "sheets" sharing the same complex coordinate. The overset region is kept to small geographical areas, as shown in panel (a) of the figure, bounded by a pair of branch point singularities located a short distance away from the overlapping tile's corner, one on each edge. The mapping function near either one of these singularities can be expanded in powers of the complex coordinate relative to the singularity using half integer powers, which causes the inverse mapping to have the two-valuedness associate with the self-overlapping Riemann surface. But the first few half-odd-integer powers can be suppressed in the construction by allowing extra compensatingly singular degrees of freedom to occur in the expanion of the function "about infinity" in the sheet corresponding to the noncomputational portion of the solution (shown as the red portions of the idealized corner detail of panels (b) and (c) of the figure).

The method of construction involves a generalization to the Riemann surface topology of the "Schwarzian" iteration, used in Rančić et al. (1996), in which each piece of the solution is described by a convergent power expansion about a nearby point, and all points along the circle of convergence of a given expansion lie well within at least one disk of convergence of another of the power expansions. Complex Fourier transformation of the diagnosed interim solution along a circle (or a circuit conformally-equivalent to a circle) just within the convergence limit enables refinement of the associated expansion coefficients, and a sequence of iterations from an initial crude guess-solution, leads towards the desired analytic solution. The only singularities – the branch points – should remain sufficiently weak to be essentially invisibe to numerics of a dynamical model core, whether it be of the finite difference or the finite volume kind. By careful reconciliation of the duplicated solutions in the small overlap regions, it is expected that grid-imprinting effects, even in extended period model runs, could be rendered insignificant.

References

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