Improved numerical solution of three-dimensional turbulence closure equations

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The results of application the atmospheric boundary layer (ABL) model /1/ in Hydro-meteorological Center (Moskow) operational prediction system /2/ revealed the effectiveness of ABL modeling if the tidy coefficients were taken suitable for use /3/.
The approach developed /4/ allowed to get the numerical integration scheme of two-equation turbulence closure for the non-stationary, one- dimensional ABL without using the tidy coefficients. The improvement and application the numerical integration scheme to the three-dimensional ABL is given in this report
The turbulent kinetic energy (TKE) and dissipation equations of three-dimensional closure scheme includes the variable sign terms of horizontal advection A, buoyancy (B) and diffusion (D) and constant sign terms of the production and dissipation. The time integration scheme is used along with the method of successive approximations.
The linear and finite-difference forms of these equations are constructed in such a way that the criteria of stability and positive numerical solution are fulfilled. It is getting in such way. The terms $A_i = \{ A, B, D \}, i = 1,2,3$ are written as

$A_i = \delta_i A_i + (1 - \delta_i) A_i$ where $\delta_i = \begin{cases} 1 & A_i \geq 0 \\ 0 & A_i < 0 \end{cases}$

The TKE (E) and dissipation (ε) equations are represented

$$\frac{\partial E}{\partial t} + \delta_1 A(E) + \delta_2 B + \delta_3 D(E) =$$

$$- (1 - \delta_1) A(E) - (1 - \delta_2) B - (1 - \delta_3) D(E)) - \alpha_\varepsilon \frac{E^2}{K} + P \qquad (1)$$

$$\frac{\partial \varepsilon}{\partial t} + \delta_1 A(\varepsilon) + \delta_2 \frac{\varepsilon}{E} B + \delta_3 D(\varepsilon) = \qquad (2)$$

$$- (1 - \delta_1) A(\varepsilon) - (1 - \delta_2) \frac{\varepsilon}{E} B - (1 - \delta_3) D(\varepsilon) + \frac{\varepsilon}{E} P - \frac{\varepsilon^2}{E}$$

The forward-differencing scheme for integration in time and the linearization of square functions $\psi^2 = \{ E^2, \varepsilon^2 \}$ with the expressions $\psi^2 = 2\psi^{n+1}\psi^n - (\psi^n)^2$ transform the TKE equation (1).

$$E^{n+1}(t + \Delta t) + \frac{E^{n+1}(t + \Delta t)}{E^n(t + \Delta t)} \times \Delta t \sum_{i=1}^{4} \delta_i A_i^n(t + \Delta t) + 2 \Delta t \times E^{n+1}(t + \Delta t) \frac{E^n(t + \Delta t)}{K^n(t + \Delta t)} =$$

$$- \Delta t \times \sum_{i=1}^{4} (1 - \delta_i) A_i^n(t + \Delta t) + \Delta t (\alpha_\varepsilon \frac{(E^n(t + \Delta t))^2}{K^n(t + \Delta t)} + P^n(t + \Delta t)) + E(t) \qquad (3)$$

The integration in time is realized in such way.

$$E^{n+1}(t+\Delta t) = (-\Delta t \times \sum_{i=1}^{4}(1-\delta_i)A_i^n(t+\Delta t) +$$
$$\alpha_\varepsilon \Delta t \frac{(E^n(t+\Delta t))^2}{K^n(t+\Delta t)} + \Delta t P^n(t+\Delta t) + E(t)) \times \tag{4}$$
$$(1 + \frac{\Delta t}{E^n(t+\Delta t)}\sum_{i=1}^{4}\delta_i A_i^n(t+\Delta t) + 2\alpha_\varepsilon \Delta t \frac{E^n(t+\Delta t)}{K^n(t+\Delta t)})^{-1}$$

The same procedure is fulfilled for the dissipation equation (2).

$$\varepsilon^{n+1}(t+\Delta t) = (-\Delta t \times ((1-\delta_1)A_1^n(t+\Delta t) + (1-\delta_5)A_5^n(t+\Delta t) +$$
$$\frac{\varepsilon^n(t+\Delta t)}{E^n(t+\Delta t)}((1-\delta_2)A_2^n(t+\Delta t) - P^n(t+\Delta t)) + \tag{5}$$
$$(1-\delta_3)A_3^n(t+\Delta t)) + \Delta t \alpha_\varepsilon \frac{(\varepsilon^n(t+\Delta t))^2}{E^n(t+\Delta t)} + \varepsilon(t)) \times$$
$$(1 + \frac{\Delta t}{\varepsilon^n(t+\delta t)}(\delta_1 A_1^n(t+\Delta t) + \delta_5 A_5^n(t+\Delta t) +$$
$$\delta_3 A_3^n(t+\Delta t)) + 2\alpha_\varepsilon \Delta t \frac{\varepsilon^n(t+\Delta t)}{E^n(t+\Delta t)} +$$
$$\frac{\Delta t}{E^n(t+\delta t)}\delta_2 A_2^n(t+\Delta t))^{-1}$$

The quantities $E^{n+1}(t+\Delta t), \varepsilon^{n+1}(t+\Delta t)$ and $E^n(t+\Delta t), \varepsilon^n(t+\Delta t)$ in the expressions (3-5) are the variables of the next and previous iterations at the prediction instance of time, $E(t), \varepsilon(t)$ are the variables at the initial instance of time period $t, t+\Delta t$. All terms in these expressions are positive and it provides the positive solution of the turbulence parameters automatically.

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