Numerical integration of two-equation turbulence closure scheme

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The developed atmospheric boundary layer modeling is based on two-equation turbulence closure, which included the turbulent kinetic energy (TKE) and dissipation equations along with Kolmogorov-Prandtl relationship. We recommended to include this more physical well-grounded approach in the prediction numerical operations as an alternative to Yamada-Mellor method and its modifications which. use TKE equation only with the set of empirical formulas.

Let's consider the non-stationary one-dimensional version of the turbulence closure when the advective terms play a secondary role in the turbulent exchange formation.

$$\frac{\partial E}{\partial t} = \eta - \mu + \alpha_E \frac{\partial}{\partial z} k \frac{\partial E}{\partial z} - \varepsilon \,, \quad \frac{\partial \varepsilon}{\partial t} = \frac{\varepsilon}{E}\left\{ a_1 \eta - a_2 \mu \right\}] - a_4 \frac{\varepsilon^2}{E} + a_3 \frac{\partial}{\partial z} k \frac{\partial \varepsilon}{\partial z}$$

$$\eta = k\left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2\right], \quad \mu = \alpha_T \frac{g}{\theta_0} k \frac{\partial \theta}{\partial z}, \quad k = \alpha_\varepsilon \frac{E^2}{\varepsilon}$$

The implicit time integration scheme is applied along with the successive sequence method. We consider the buoyancy and dissipation terms in the form which provide to fulfill the conditions of numerical stability and positive solution. TKE and dissipation equations are represented with the following expressions:

$$\frac{E_i^{m+1}(t+\delta t) - E_i(t)}{\delta t} = \eta_i^m + \alpha_E \left(\frac{\partial}{\partial z} k \frac{\partial E}{\partial z}^{m+1}\right)_i - \alpha_\varepsilon \left(2 E_i^{m+1} E_i^m - (E_i^m)^2\right) / k_i^m - \quad (\frac{E_i^{m+1}}{E_i^m} \delta \mu_i^m + (1-\delta)\mu_i^m)$$

$$\frac{\varepsilon_i^{m+1}(t+\delta t) - \varepsilon_i(t)}{\delta t} = a_1 \frac{\varepsilon_i^m}{E_i^m} \eta_i^m + a_3 \left(\frac{\partial}{\partial z} k \frac{\partial \varepsilon_i^{m+1}}{\partial z}\right)_i - a_4 \frac{2\varepsilon_i^{m+1}\varepsilon_i^m - (\varepsilon_i^m)^2}{E_i^m} - (\frac{\varepsilon_i^{m+1}}{E_i^m}\delta\mu_i^m + \frac{\varepsilon_i^m}{E_i^m}(1-\delta)\mu_i^m)$$

$$\mu \ge 0, \delta = 1, \mu < 0, \delta = 0$$

We developed three approaches of numerical integration of two-equation turbulence closure to restore ABL vertical structure.

.1. The equations of the variables $E_i^{m+1}, \varepsilon_i^{m+1}$ in the instance $t + \delta t$ are presented in under mentioned expressions.

$$E_i^{m+1} = (D_i + d_{i+1} E_{i+1}^m + d_{i-1} E_{i-1}^m) / d_i \,, \quad \varepsilon_i^{m+1} = (C_i + c_{i+1}\varepsilon_{i+1}^m + c_{i-1}\varepsilon_{i-1}^m) / c_i$$

2. We find the solution of equations () separately with the method of factorization.

$$d_{i+1} E_{i+1}^{m+1} - d_i E_i^{m+1} + d_{i-1} E_{i-1}^{m+1} = -D_i$$

$$d_{i+1} = \alpha_E \frac{k_{i+1}^m + k_i^m}{2(\delta z)^2}, d_{i-1} = \alpha_E \frac{k_i^m + k_{i-1}^m}{2(\delta z)^2}$$

$$d_i = 1/\delta t + d_{i+1} + d_{i-1} + \delta\mu_i^m / E_i^m + 2\alpha_\varepsilon E_i^m / k_i^m$$

$$D_i = E_i(t)/\delta t + \eta_i^m - (1-\delta)\mu_i^m + \alpha_\varepsilon (E_i^m)^2 / k_i^m$$

$$c_{i+1}\varepsilon_{i+1}^{m+1} - c_i \varepsilon_i^{m+1} + c_{i-1}\varepsilon_{i-1}^{m+1} = -C_i$$

$$c_{i+1} = a_3 \frac{k_{i+1}^m + k_i^m}{2(\delta z)^2}, c_{i-1} = a_3 \frac{k_i^m + k_{i-1}^m}{2(\delta z)^2}$$

$$c_i = 1/\delta t + c_{i+1} + c_{i-1} + a_2 \delta\mu_i^m / E_i^m + 2a_4 \varepsilon_i^m / E_i^m$$

$$C_i = \varepsilon_i(t)/\delta t + a_1 \frac{\varepsilon_i^m}{E_i^m}\eta_i^m - a_1 \frac{\varepsilon_i^m}{E_i^m}(1-\delta)\mu_i^m + \alpha_\varepsilon (\varepsilon_i^m)^2 / E_i^m$$

3. We solved the two-equation closure system with the matrix factorization method

$$d_{i+1} E_{i+1}^{m+1} - d_i E_i^{m+1} + d_{i-1} E_{i-1}^{m+1} - \varepsilon_i^{m+1} = -D_i$$

$$c_{i+1}\varepsilon_{i+1}^{m+1} - c_i \varepsilon_i^{m+1} + c_{i-1}\varepsilon_{i-1}^{m+1} - a_4 \frac{\alpha_\varepsilon}{k_i^m} E_i^{m+1} = -C_i$$

$$d_{i+1} = \frac{k_{i+1}^m + k_i^m}{2(\delta z)^2}, d_{i-1} = \frac{k_i^m + k_{i-1}^m}{2(\delta z)^2}$$

$$d_i = 1/\delta t + d_{i+1} + d_{i-1} + \delta\mu_i^m / E_i^m$$

$$D_i = E_i(t)/\delta t + \eta_i^m - (1-\delta)\mu_i^m$$

$$c_{i+1} = a_3 \frac{k_{i+1}^m + k_i^m}{2(\delta z)^2}, c_{i-1} = a_3 \frac{k_i^m + k_{i-1}^m}{2(\delta z)^2}$$

$$c_i = 1/\delta t + c_{i+1} + c_{i-1} + a_2 \delta\mu_i^m / E_i^m$$

$$C_i = \varepsilon_i(t)/\delta t + a_1 \frac{\varepsilon_i^m}{E_i^m}\eta_i^m - a_1 \frac{\varepsilon_i^m}{E_i^m}(1-\delta)\mu_i^m$$

The generalization of aforementioned approaches can be obtained if the advection terms are included in the TKE and dissipation equations.

Some assumptions about the application of developed approaches are suggested. The first approach can be used when the parameterization of ABL is applied in large scale prediction models where the horizontal step is much more than the ABL depth and is equal 50 km or more. The third approach can be applied when the horizontal step is comparable with the ABL depth which isn't more than 5 km. The second approach is acceptable for the intermediate horizontal scales. But it's the assumptions only and the solution of this problem is in question.