Forth Order Compact Difference Scheme for Horizontal Block of Forecasting Model.

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The typical computational problem in the horizontal block of global forecasting models with implicit or semi-implicit approximation is the following: to restore the horizontal wind by its divergence D and vorticity ζ on the sphere. The problem is solved for every vertical level and on every time step, and therefore its exactness and effectiveness are very important.

Fast Fourier transform $F: \quad f(\varphi,\lambda) \to \hat{f}(\varphi,m)$ with respect to longitude is a natural approach, but then we obtain a system of ordinary differential equations with variable coefficients and special boundary conditions that both depend on the m. These boundary conditions state that several derivatives of $\hat{f}$ must be equal to zero for original function f to be smooth.

This complex system of differential equations can be separated into 2 independent real systems of differential equations. These systems can be numerically solved by the same algorithm.

We present now a compact scheme with 4-th approximation order for the problem. The equation (all functions here are after Fourier transform) $D = \frac{m}{\sin\varphi}u + v\,\mathrm{ctg}\varphi + \frac{\partial v}{\partial\varphi}$ is approximated as

$$\frac{1}{6}D_{i-1} + \frac{2}{3}D_i + \frac{1}{6}D_{i+1} = \frac{m}{6\sin\varphi_{i-1}}u_{i-1} + \frac{2m}{3\sin\varphi_i}u_i + \frac{m}{6\sin\varphi_{i+1}}u_{i+1} +$$

$$+\left[\frac{1}{2h} + \frac{\mathrm{ctg}\varphi_{i+1}}{6}\right]v_{i+1} + \frac{2\mathrm{ctg}\varphi_i}{3}v_i + \left[-\frac{1}{2h} + \frac{\mathrm{ctg}\varphi_{i-1}}{6}\right]v_{i-1},\ i = 2,\ldots,N-2$$

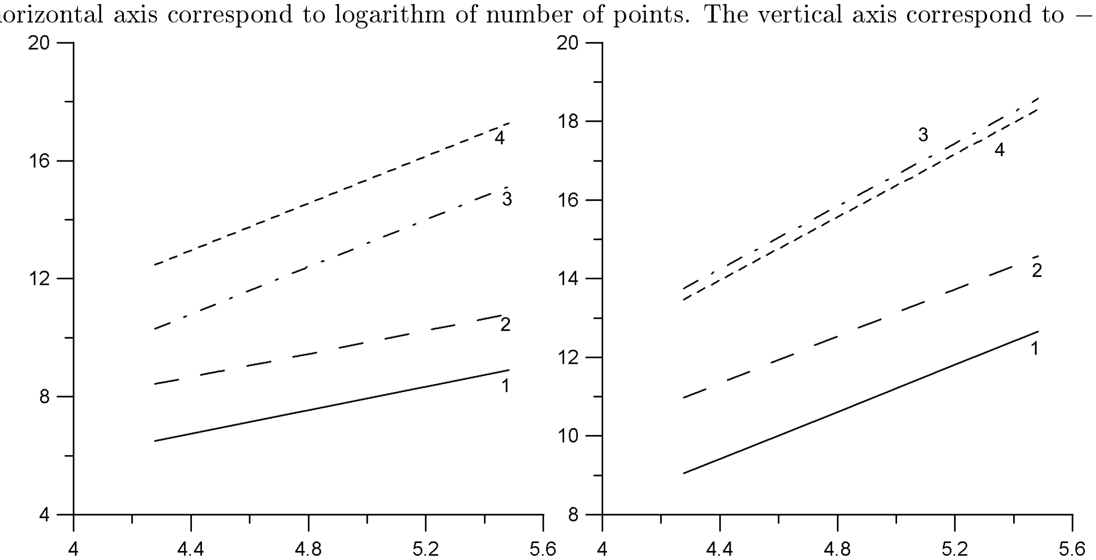
Similar approximation is used for equation $\zeta = -\frac{m}{\sin\varphi}v - u\,\mathrm{ctg}\varphi - \frac{\partial u}{\partial\varphi}$. These approximations are not suitable in boundary points due to division by zero. In this case we solve the system of linear equations on coefficients of approximation taking boundary conditions for u, v, D and ζ into account. Then the system of linear equation on u_i and v_i is solved by the sweep method and reverse Fourier transformation is applied.

Our experiments confirmed the announced approximation order. For instance, if

$$u = -\cos\varphi(3\cos^2\varphi - 3\sin^2\varphi)\sin\lambda - 0.5\sin\varphi, \quad v = \cos^2\varphi\cos\lambda,$$

$$\zeta = \sin\varphi(16\sin^2\varphi - 13)\sin\lambda + \cos\varphi, \quad D = \cos\lambda\sin\varphi\cos\varphi,$$

Next figures show errors of restoring u and v. Graphics 1 and 2 correspond to the error in u and v when using method from [4]. Graphics 3 and 4 correspond to the error in u and v when using our method. The horizontal axis correspond to logarithm of number of points. The vertical axis correspond to $-\log(\text{error})$.



Left figure shows error in **C** metric, while right figure shows errors in $\mathbf{L}_2$ metric.

References

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