On expansion of analytical functions in ultraspherical polynomials on a sphere.

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A new algorithm is suggested for the approximation of an analytical function by double Fourier series on a sphere using a complete set of ultraspherical polynomials as orthogonal basis functions. The Legendre and Chebyshev polynomials of the first and second kinds, which are special cases of the ultraspherical polynomial family, are considered.

The series obtained with the new algorithm uniformly converge in all points of a sphere, including poles. In contrast to traditional spectral series on a sphere, these series explicitly include some additional terms, characterizing an odd (with respect to poles) component of the analytically approximated function.

It is shown that in a small vicinity of poles the resulting double Fourier series becomes simpler because some Fourier terms, responsible for approximation of the odd components of the function, tend to zero in this region.

At the same time, the contribution of the asymmetric (with respect to the pole) components of the approximated function to the Fourier series grows toward the equator and becomes comparable with that of the symmetric components in the equatorial band.

The algorithm was implemented on a computer, and numerical experiments were performed, in which a given scalar continuous analytical function was expanded using spherical harmonics as an orthogonal basis.

The resulting double Fourier series turned out to be longer than the series obtained by any traditional spectral method thus better approximating the given function.

The study was supported by the Russian Foundation for Basic Research (project no. 07-05-13607-ofi_ts).