About the formation of a cloud of small vortices in the case of the interaction of a few larger vortices of finite dimensions.

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Numerical modeling (Pokhil et al. 1991-2006) shows that the interaction of vortices is accompanied by the formation of secondary smaller vortices. Let's estimate approximately the number and the total mass of the obtained small vortices reasoning from the laws of conservation of mass, the moment of momentum and energy.

In real atmospheric vortices the Reynolds number is great ($Re>10^7$). We will assume that the energy dissipation during the interaction of vortices that is studied here can be neglected. The vortex effective mass is proportional to ρR^2, the moment of momentum – to ρR^3V, the energy – to ρR^2V^2 (ρ is the density). The calculation of the energy and moment of momentum reduces to the calculation of convergent integrals in the case of a sufficiently strong function of the wind velocity decrease at $r>R$, e.g. the exponential one. The vortex effective mass will be considered as the mass of the area that contributes to the vortex moment of momentum most (for example, 0.9). We will also assume that all the vortices have identical wind velocity profile $V(r)/V=f(r/R)$. If n_1 identical vortices with the radius R_1 draw together and as a result it remains (or there forms) n_2 identical vortices with identical rotation and with the radius R_2, and also it forms n_3 small vortices with the radius R_3, the conservation laws take the form

$$\begin{aligned} &n_1R_1^2=n_2R_2^2+n_3R_3^2\\ &n_1R_1^3V_1=n_2R_2^3V_2-n_3R_3^3V_3\\ &n_1R_1^2V_1^2=n_2R_2^2V_2^2+n_3R_3^2V_3^2 \end{aligned} \qquad (1)$$

It is assumed that the velocity of the vortices' motion is substantially less than the velocity of wind V in them and that the radius of small vortices is substantially less than that of the main vortices.

1. If the wind velocities in the small vortices are little, these vortices contribute to the overall energy and to the moment of momentum only slightly. Taking this into account we obtain that the relative mass m_3/m_1 of the vortices that have formed vs. the mass of the initial vortices and their relative number n_3/n_1 take the form

$$m_3/m_1= 1-(n_2/n_1)^{1/2};\quad n_3/n_1=(1-(n_2/n_1)^{1/2})(R_1/R_3)^2 . \qquad (2)$$

The radius R_2 and the wind velocity V_2 of the newly formed great vortices just slightly depends on the number of these vortices, because $R_2/R_1 = V_2/V_1 = (n_1/n_2)^{1/4}$. The relative mass of the "great" vortices that have formed is then

$$m_2/m_1= n_2R_2^2/n_1R_1^2=(n_2/n_1)^{1/2}.$$

It is easy to see that we obtain a positive solution only in the case of $n_2/n_1<1$. When $n_2=2$ and $n_2=1$, 0.3 of the mass of initial vortices passes into small vortices.

2. If $V_2/V_1=1$, then small vortices contribute to the overall energy to the extent comparable with the full energy, and to the moment of momentum they contribute to a small extent, because $R_3<<R_1$. In this case we obtain

$$\begin{aligned} &m_3/m_1=1-(n_2/n_1)^{1/3};\quad n_3/n_1=(1-(n_2/n_1)^{1/3})(R_1/R_3)^2\\ &V_2/V_1=1;\quad R_2/R_1=(n_1/n_2)^{1/3} \end{aligned} \qquad (3)$$

At $n_1=2$ and $n_2=1$ the mass of small vortices is equal to ~0.2 of the mass of the initial vortices.

Thus, the overall mass of small vortices is just slightly dependent on the maximum wind velocity in them. Furthermore, all the presented results practically do not depend on the sign of the small vortices and remain unchanged if there are small vortices of different signs as well.

3. Suppose that, while interacting, several like vortices split into two groups of small vortices, with one of them forming a "compound" central vortex consisting of a multitude of small vortices and the second one forming a cloud of comparatively slow-moving vortices as compared to the maximum wind velocity in the initial vortices (Pokhil et al. 1991-1997). Let's assume that in the central vortex that has formed small vortices are distributed uniformly, so that the mean density of matter in the "compound" vortex is constant and $\rho_2<\rho_1$, where ρ_1 is the matter density in the initial vortices, ρ_2 is that in the resulting compound vortices. Furthermore, let's suppose that the wind velocity in all the small vortices and the velocity of any motion of the vortices of the second group are considerably less than the maximum wind velocity in the resulting "compound" vortices of the second group. On these assumptions the conservation equations remain the same as in the second case, if in these equations we change n_2 for $n_2\cdot\rho_2/\rho_1$. So, the radius of the obtained compound vortices is R_2, the maximum velocity in them is V_2 and the relative mass and the number of small vortices will be equal to

$$m_3/m_1=1-\sqrt{(n_2\rho_2)/(n_1\rho_1)}\quad;\quad R_2/R_1=V_2/V_1=(n_1\rho_1/n_2\rho_2)^{1/4} \qquad (4)$$

$$n_3/n_1=(1-\sqrt{n_2\rho_2\cdot n_1\rho_1})(R_1/R_3)^2\;;$$

Here, as in the previous case that we have considered, n_1 is the number of initial vortices, n_3 is the number of small vortices of the group, R_3 is their radius.

In the case under consideration we obtain positive solutions at $n_2\rho_2/n_1\rho_1<1$, i.e. it is allowed that the number of the obtained compound vortices would be equal or would even exceed the number of initial continuous vortices. If out of two initial vortices one "compound" vortex forms, its mass constitutes a minor part of the mass of initial vortices.

It should be noted that the effective density ρ_2 of the compound vortex has a minor effect on its diameter and velocity.

4. It was supposed above that the wind velocity profile in the newly formed and initial vortices is identical. If the profile of the wind velocities does not change while identical vortices with identical rotation interact, a cloud of small vortices ($R_3/R_1<<1$) can form only if after the interaction the number of identical vortices formed after the interaction is less than that of the initial vortices. Nevertheless, numerical modeling shows that in the case of the interaction of two identical vortices with identical rotation both vortices can remain unchanged, forming at the same time a cloud of small vortices (Pokhil et al. 1991-2006).

In connection with such observations let's examine the interaction n_1 of identical vortices with identical rotation, from which results the formation of "great" and "small" vortices, with the wind velocity profile in the vortices becoming other than before the interaction. The interaction results in the formation of n_2 "large" vortices comparable with the initial ones, and in the formation of n_3 "minor" vortices whose radius is less than that of the initial vortices. Let's assume that the small vortices are characterized by a little velocity $V_3/V_1<<1$. Suppose that the mass, the moment of momentum and the energy of the initial and the newly formed vortices, taking into account the change of the wind profile after the interaction, are as follows:

mass – $\alpha_i \rho R_i^2$

the moment of momentum – $\beta_i \rho R_i^3 V_i$ (5)

energy – $\gamma_i \rho R_i^2 V_i^2$, where α_i, β_i, γ_i depend on the distribution of the wind velocities.

Then the conservation laws at $R_3/R_1<<1$ and $V_3/V_1<<1$ take the form:

$$\alpha_1 n_1 R_1^2 = \alpha_2 n_2 R_2^2 + \alpha_3 n_3 R_3^2 \quad (6)$$
$$\beta_1 n_1 R_1^3 V_1 = \beta_2 n_2 R_2^3 V_2$$
$$\gamma_1 n_1 R_1^2 V_1^2 = \gamma_2 n_2 R_2^2 V_2^2$$

Having solved the equations we will obtain that the relative masses of the originated "large" and "minor" vortices are equal to:

$$m_2/m_1 = (\alpha_2/\alpha_1)(\beta_1/\beta_2)(\gamma_2/\gamma_1)^{½}(n_2/n_1)^{½} \quad \text{и}$$
$$m_3/m_1 = 1-(\alpha_2/\alpha_1)(\beta_1/\beta_2)(\gamma_2/\gamma_1)^{½}(n_2/n_1)^{½}; \quad (7)$$

If the coefficient $(\alpha_2/\alpha_1)(\beta_1/\beta_2)(\gamma_2/\gamma_1)^{½}<1$, then the cloud of small vortices can form at $n_3/n_1=1$, i.e. if the number of originated "large" vortices is the same as in the initial case.

Such a situation occurs if the originating "large" vortices are characterized by a more sharp velocity drop than the initial vortices.

5. If it is the case of the interaction of vortices with different rotation, whose total the moment of momentum is equal to zero, the conservation laws restrict the parameters of the originated vortices to a lesser extent than in the case of the interaction of the vortices with identical rotation.

In case if slow-moving ($V_3/V_1<<1$) small vortices are formed, the conservation laws are as in (6); with the difference that the total the moment of momentum is equal to zero.

Let's consider a most simple case when out of two vortices with different rotation form two vortices also with different rotation, with the characteristic radii of all the four vortices being equal, and, besides that, the interaction results in the formation of small vortices with $R_3/R_1<<1$ and $V_3/V_1<<1$. In this case

$$2\alpha_1 R_1^2 = 2\alpha_2 R_2^2 + n_3 \alpha_3 R_3^2$$

Taking into account that, by the data, , we obtain

$$m_2/m_1 = (\alpha_2/\alpha_1)(R_2/R_1)^2 = \alpha_2/\alpha_1 \quad \text{и} \quad (8)$$
$$m_3/m_1 = (\alpha_3/\alpha_1)(n_3/n_1)(R_3/R_1)^2 = 1-\alpha_2/\alpha_1, \quad (9)$$

that is, a cloud of small vortices forms when the velocity drop becomes sharper.

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