Improved numerical scheme of turbulence closure.

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We developed an improved numerical scheme for solution of turbulence closure equations and demonstrate its effectiveness applying it to the one-dimensional non-stationary atmospheric boundary layer (ABL). The two-equation closure scheme includes the equations of turbulent kinetic energy (1) and dissipation rate (2) along with Kolmogorov-Prandtl relationship for the turbulence coefficient (3) [Shnaydman, Berkovich].

$$\frac{\partial E}{\partial t} = k\left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2\right] - \alpha_T k \frac{g}{\theta_0}\frac{\partial \theta}{\partial z} + \alpha_\varepsilon \frac{\partial}{\partial z} k \frac{\partial E}{\partial z} - \varepsilon \qquad (1)$$

$$\frac{\partial \varepsilon}{\partial t} = \frac{\varepsilon}{E}\left\{\alpha_1 k\left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2\right] - \alpha_2 k \frac{g}{\theta_0}\frac{\partial \theta}{\partial z} - \alpha_3 \varepsilon\right\} + \alpha_4 \frac{\partial}{\partial z} k \frac{\partial \varepsilon}{\partial z} \qquad (2)$$

$$k = \alpha_\varepsilon \frac{E^2}{\varepsilon} \qquad (3)$$

Nowadays the two-equation turbulence closure became a standard feature of many ABL models. So it is important to develop adequate numerical algorithms for solution equations (1-3). This algorithm has to be numerically stable for relatively large time steps and positively defined for turbulence kinetic energy (TKE) and dissipation. Unfortunately in many ABL models the fulfillment of these requirements depends on relations between the mechanisms of ABL formation especially between the TKE production and the effect of the buoyancy force [Jiang]. This in some cases could produce erroneous results. Therefore here we developed a finite-difference scheme for (1-3) that is numerically stable and keeps TKE and ε positive throughout entire integration.

We conducted numerical experiments to choose the most suitable form for the non-linear and buoyancy terms. First we realized that linearization of square terms on one time step has to be done in the following way ($\varphi = (E, \varepsilon)$, φ and φ^n are the values of unknown variables at given time t and at the previous time step):

$$\varphi^2 = 2\varphi \times \varphi^n - (\varphi^n)^2 \qquad (4)$$

Then we multiply the buoyancy term by δ for stable stratification when δ =1 and by 1-δ for unstable stratification when δ =0.

Using these relations we rewrite the TKE and dissipation equations in the following form:

$$\frac{\partial E}{\partial t} + \left(2\alpha_T \alpha_\varepsilon \delta \times \frac{g}{\theta_0}\frac{\partial \theta}{\partial z} \times \frac{E^n}{\varepsilon^n}\right) \times E - \frac{\partial}{\partial z} k \frac{\partial E}{\partial z} + \varepsilon = F_E^n \tag{5}$$

$$\frac{\partial \varepsilon}{\partial t} + \left(2\alpha_2 \alpha_T \alpha_\varepsilon \delta \times \frac{g}{\theta_0}\frac{\partial \theta}{\partial z}\right) \times E - 2\alpha_3 \frac{\varepsilon^n}{E_n} \times \varepsilon - \alpha_4 \frac{\partial}{\partial z} k \frac{\partial \varepsilon}{\partial z} = F_\varepsilon^n \tag{6}$$

$$F_E^n = k_n \left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2 + (2\delta - 1)\alpha_T \frac{g}{\theta_0}\frac{\partial \theta}{\partial z} \right] \tag{7}$$

$$F_\varepsilon^n = \alpha_\varepsilon E^n \left[\alpha_1 \left(\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2 \right) - (1 - \delta)\alpha_2 \frac{g}{\theta_0}\frac{\partial \theta}{\partial z} \right] + \alpha_3 \frac{(\varepsilon^n)^2}{E^n} \tag{8}$$

The finite difference equations were obtained using first-order approximation in time, and centered-in-space differences for the vertical turbulent terms (second-order approximation in space). The implicit numerical integration scheme was applied. This scheme is used here for the stationary problem:

$$a_1 E_{m-1} - a_2 E_m + a_3 E_{m+1} + \varepsilon_m = (F_E^n)_m \delta t + E_m^n \tag{9}$$

$$b_1 \varepsilon_{m-1} - b_2 \varepsilon_m + b_3 \varepsilon_{m+1} + b_4 E_m = (F_\varepsilon^n)_m \delta t + \varepsilon_m^n \tag{10}$$

Now we can re-write the system (9-10) in matrix form:

$$A W_{m-1} - B W_m + C W_{v+1} = D_m \tag{11}$$

Equation (11) was solved numerically using factorization method. The conditions of stability and positive solution were fulfilled independently and turbulent kinetic energy and dissipation rate were kept positive for all conditions.

Reference

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