A covariance model based on 3-D spatial filters: potential for flow-dependent covariance modelling

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1 Introduction

In data assimilation, an advanced forecast (background) error covariance model is required to be capable of representing spatially variable and flow-dependent structures. These involve inhomogeneities in variances and decorrelation lengths: in the vertical, over latitude, between sea and land, dependent on the weather system etc. We also wish to model local horizontal anisotropies due to fronts, jet streams etc. Finally, vertically tilted structures typical for baroclinic zones are to be represented in the covariance model.

In this short contribution, we present a covariance model being developed in the Hydrometcentre of Russia and demonstrate its capability of modelling spatially variable structures. The univariate aspect is considered.

2 The covariance model

The covariance (stochastic) model is of the spatial auto-regression and moving-average (SARMA) type:

$$S\xi = V\alpha, \tag{1}$$

where ξ is the background-error field, α the driving white noise, S the spatial auto-regression linear filter, and V the spatial moving-average filter. Each of the two operators is defined by using discretized integral or differential operators (see below), giving rise to a *sparse* (and thus computationally efficient) matrix formulation.

In order to specify the S and V operators in Eq.(1) we propose the following construction. Because the vertical direction is the very special direction in the Earth's atmosphere (and ocean), we define S and V such that Eq.(1) is a *one-dimensional ARMA model in the vertical*, so that model Eq.(1) becomes

$$P_S(\partial/\partial z) \cdot \xi = P_V(\partial/\partial z) \cdot \alpha, \tag{2}$$

where z is the vertical coordinate, P_S and P_V are the polynomials whose coefficients are *horizontal operators*.

Examination of the 3-D ECMWF background-error covariances in (Tsyroulnikov 2001) suggested that (spectral) vertical correlations can be modelled with the Kagan's or degenerate 3-rd order auto-regression model. In view of this finding, we simplify the above model by defining P_S to be $(\partial/\partial z + T)^q$, where $q = 3$ and T is the horizontal operator. We define P_V to be a zero-order polynomial, $P_V = U$, where U is the horizontal operator. Thus, our stochastic model reads

$$(\frac{\partial}{\partial z} + T)^q \cdot \xi = U \cdot \alpha. \tag{3}$$

We have to synthesize the horizontal operators T and U that produce the desired 3-D covariance structure and also lead to a computationally efficient analysis algorithm. As for T, it appears to be possible to approximate it using a (very fast) finite-difference operator:

$$T = P_T(-\hat{\Delta}), \tag{4}$$

where P_T is the low-order polynomial and $\hat{\Delta}$ is a finite-difference approximation to the horizontal Laplacian. As for U, it appears to be more reasonable to approximate it with the discretized integral operator:

$$(U \cdot \alpha)(x) = \int u(\rho(x,y))\alpha(y)dy, \tag{5}$$

where x and y are points in the horizontal (spherical or plane) domain, ρ the distance between x and y, and $u(\rho)$ is the function that has *small* support to enforce sparse matrix algebra.

The above definition of the SARMA model is based on *operators* and thus is essentially coordinate-free, hence its applicability on any domain in any geometry. The model produces fully non-separable 3-D correlations.

3 Spatially variable covariances

Spatial variability can be introduced into model Eq.(3) by specifying spatially variable operators T and U. E.g., in Fig.1 we show a realization of the pseudo-random field (the horizontal cross-section) generated with model Eq.(3) in which the horizontal scale of $u(\rho)$ was intentionally decreased in a 'cyclone' located at 45N, 0E.

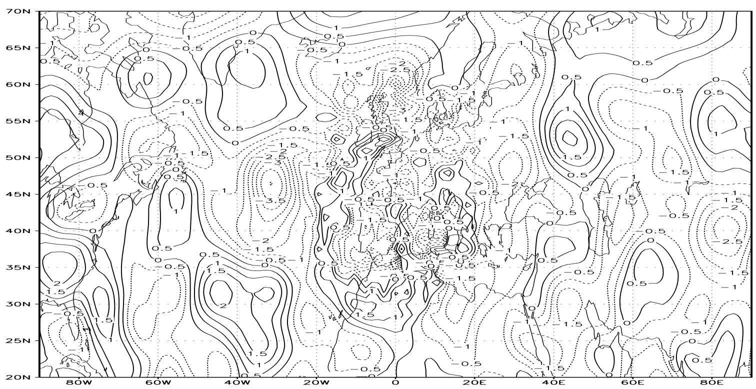

Figure 1: Horizontal inhomogeneity in the horizontal length scale. (the lat-lon cross-section)

Tilted structures are modelled by adding, in Eq.(3), to T, the term $\mathbf{c}\nabla$ (where $\mathbf{c}$ is a horizontally variable horizontal vector and ∇ the horizontal gradient operator). The resulting pseudo-random field (for the case when $\mathbf{c}$ is non-zero in the 'cyclone') looks as in Fig.2 (the vertical cross-section).

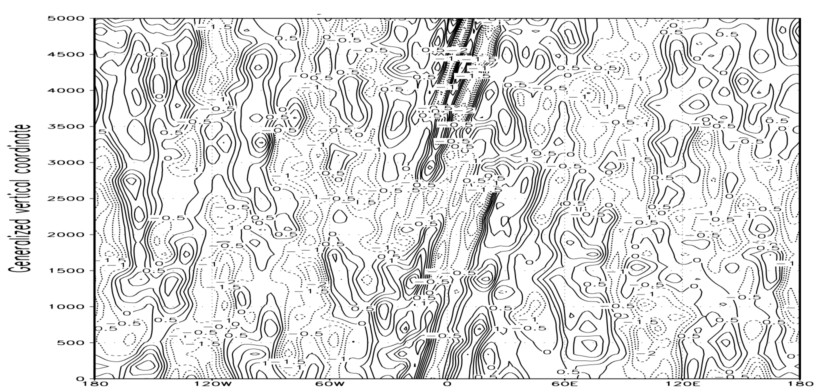

Figure 2: Modelling tilted structures (the longitude-height cross-section)

We would stress that any spatial (in particular, flow-dependent) variability introduced to model Eq.(3) cannot, by construction, violate positive definiteness of the resulting covariance matrices.

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