Development of a 3-D spatial ARMA-filters based analysis scheme

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1 Introduction and general design

A new 3-D meteorological data assimilation scheme is being developed in the RHMC. The principal features required of the scheme are:

(1) The scheme is intended to be as universal as possible. First, it should be applicable for data assimilation on all spatial scales (global, regional, and meso-scale). Second, the core of the scheme is to be usable for ocean data assimilation.

(2) The analysis scheme should be flexible enough to efficiently utilize spatially variable flow-dependent background-error covariances expected from a future Ensemble Kalman Filter.

The new 3D-Var type scheme relies on 3-D physical-space spatial filters. The analysis equations are solved in model (grid) space using a preconditioned conjugate-gradient solver. The scheme utilizes spherical geometry in the horizontal and a hybrid coordinate in the vertical.

2 The covariance model

To obtain flexibility while ensuring positive definiteness, we define the spatial covariance model *constructively*, i.e. we build a model for the underlying random field(s). Another important issue to be addressed is computational efficiency, so we would like the covariance model to produce compactly (locally) supported correlation (or related) functions. These two features (constructive definition and local support) has led us to introduce a spatial generalization of the well-known—in the time-series theory and practice—ARMA (auto-regression moving average) model. In the one-dimensional case, ARMA models proved to be very efficient in modelling many realistic random processes, so that in most situations the orders of both AR and MA digital filters estimated from real data appeared to be very small. Small orders imply small supports of the impulse response functions for the respective filters, hence, both AR and MA operators can be represented by very sparse matrices. This is of primary importance for our practical data assimilation system in view of huge dimensionality of atmospheric and oceanic data assimilation problems. The resulting SARMA (spatial ARMA) model writes in the space-continuous form:

$$S\xi = V\alpha, \tag{1}$$

where ξ is the background-error field, α denotes the white noise, S is the SAR filter (linear integral operator), and V is the SMA filter.

In this article, we consider the 2-D univariate problem on the sphere S^2. We develop an isotropic model, which is to be used as a building block in the future real implementation. Isotropy implies that each of the two operators, S and V, in Eq.(1) is defined by its generating radial function, $s(\rho)$ and $v(\rho)$, respectively, where ρ denotes the angular distance, e.g. for S:

$$(S\xi)(x) = \int_{S^2} s(\rho(x,y))\xi(y)dy, \tag{2}$$

where x and y are points on the sphere and dy the area element. In order to use the above SARMA model in the analysis scheme, we discretize the supports of the random fields ξ and α, getting

$$\mathbf{S}\vec{\xi} = \mathbf{V}\vec{\alpha}, \tag{3}$$

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where $\mathbf{S}$ and $\mathbf{V}$ matrices are easily derived from the generating functions $s(\rho)$ and $v(\rho)$ and $\vec{\alpha}$ is the unit-variance white-noise vector. Hence, the background-error covariance matrix is

$$\mathbf{B}_\xi = \mathbf{W}\mathbf{W}^*, \qquad \text{where} \qquad \mathbf{W} = \mathbf{S}^{-1}\mathbf{V}, \tag{4}$$

The great advantage of the above *constructive* model formulation is that we are free to change matrices $\mathbf{S}$ and $\mathbf{V}$ in any way (to account for spatially variable background-error statistics) without a danger to lose well-posedness of the model.

3 Estimation of the SARMA model

Having an estimate, $B^{emp}(\rho)$, of the true covariance function, $B(\rho)$, we seek functions $s(\rho)$ and $v(\rho)$ such that: (i) the implied model covariance function is close to the empirical one, and (ii) both s and v have as small supports as possible. A variational formulation is used to solve this problem. Some results for two correlation functions estimated at DWD (Anlauf et al.2005) are shown in Fig.1.

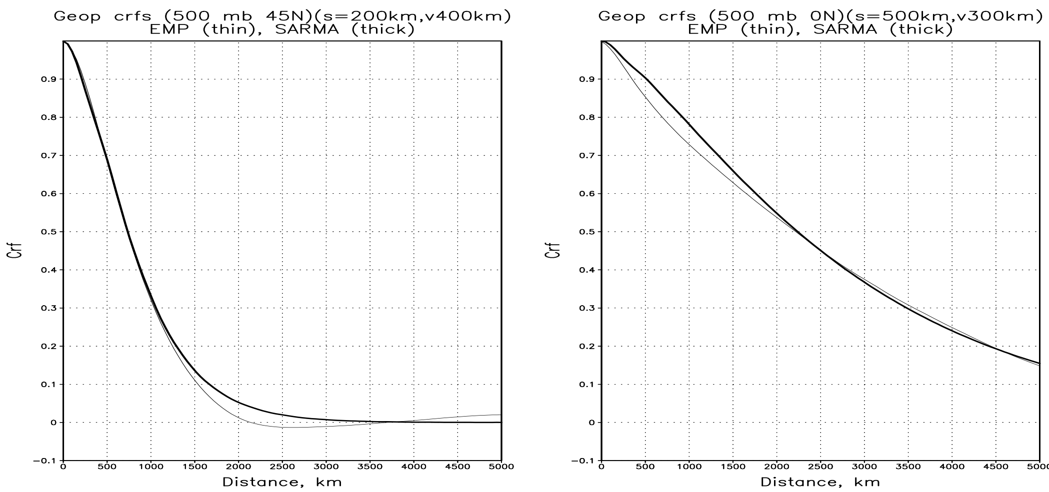

Figure 1: Empirical (thick) and model (thin) correlation functions for 500-hPa geopotential errors. *Left*: Latitude 45N. Supp(s)=200 km, Supp(v)=400 km. *Right*: Latitude 0. Supp(s)=500 km, Supp(v)=300 km.

One can see that very small supports ($\leq$ 500 km) of functions s and v appeared to be sufficient to accurately approximate quite broad correlations. Small supports imply that both S and V matrices are very sparse, which is of paramount importance for the computational analysis algorithm.

4 Conclusions

The new 3D-Var type assimilation scheme is under development in the Russian Hydrometcentre. The scheme is based on the spatial ARMA filters. Work on the numerical solver and the observational (in situ and satellite) processing scheme is in progress. Currently, a 2-D univariate version of the scheme is tested with synthetic observations.

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References

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