Entropy production of the oceanic general circulation

Shinya Shimokawa * and Hisashi Ozawa **
* National Research Institute for Earth Science and Disaster Prevention, Tsukuba 305-0006, Japan (simokawa@bosai.go.jp)
** Hiroshima University, Higashi-Hiroshima 739-8521, Japan

1. Introduction

Ocean system is regarded as an open dissipative system connected to the surrounding system (atmosphere and the universe) mainly through heat and salt fluxes. In this view point, the formation of a circulatory structure can be regarded as a process leading to the final equilibrium of the whole system consisting of the ocean system and the surrounding system. In this process, the rate of approach to the equilibrium, i.e. the rate of entropy production by the oceanic circulation, seems to be an important factor. We have so far investigated the relationship between the *global* (total) entropy production in the ocean system and the formation of the circulatory structure (Shimokawa & Ozawa, 2001, 2002, 2005). In addition, we are also interested in the *local* (distribution of) entropy production in the ocean system to obtain a complete understanding of the ocean system. The objective of this study is to evaluate the local rate of entropy production in the ocean system by using an oceanic general circulation model.

2. Formulation of entropy production in the ocean system

The global rate of entropy production $\dot{S}$ is calculated in the ocean system such as

$$\dot{S} = \int \frac{\rho c}{T}\frac{\partial T}{\partial t}\, dV + \int \frac{F_h}{T}\, dA - \alpha k \int \frac{\partial C}{\partial t} \ln C\, dV - \alpha k \int F_s \ln C\, dA. \qquad (1)$$

where ρ is the density, c is the specific heat at constant volume, T is the temperature, $\alpha=2$ is van't Hoff's factor representing the dissociation effect of salt into separate ions (Na^+ and Cl^-), k is the Boltzmann constant, C is the number concentration of salt per unit volume of sea water, F_h and F_s are the heat and salt fluxes per unit surface area, defined as positive outward, respectively. The first term on the right hand side represents the rate of entropy increase in the ocean system due to heat transport, and the second term represents that in the surrounding system. The third term represents the rate of entropy increase in the ocean system due to salt transport, and the fourth term represents that in the surrounding system. Overall, Equation (1) represents the rate of entropy of the whole system, i.e. the entropy production due to irreversible process associated with the oceanic circulation.

This expression can be rewritten in a different form with some mathematical transformation such as

$$\dot{S} = \int \boldsymbol{F_h}\cdot \mathbf{grad}\left(\frac{1}{T}\right) dV + \int \frac{\Psi}{T}\, dV - \cdot k \int \frac{\boldsymbol{F_s}\cdot \mathbf{grad}\, C}{C}\, dV, \qquad (2)$$

where $\boldsymbol{F_h}$ and $\boldsymbol{F_s}$ are the flux density of heat and salt (vector in three dimensional space), respectively, and Ψ is the dissipation function, representing the rate of dissipation of kinetic energy into heat by viscosity per unit volume of the fluid. The first term on the right-hand side is the entropy production rate by thermal dissipation (heat conduction), the second term is that by viscous dissipation, and the third term is that by molecular diffusion of salt ions.

Since entropy production due to salt transport is negligible for the ocean system (Shimokawa & Ozawa, 2001), the local entropy production can be estimated from the first term in (2) such as

$$A = \frac{\rho c}{T^2}(A_x+A_y+A_z), \quad A_x = D_h\left(\frac{dT}{dx}\right)^2, \quad A_y = D_h\left(\frac{dT}{dy}\right)^2, \quad A_z = D_v\left(\frac{dT}{dz}\right)^2, \qquad (3)$$

where D_h is the horizontal diffusivity, D_v is the vertical diffusivity. It is assumed here that $F_h = -k\,\mathrm{grad}(T) = -\rho c D_E\,\mathrm{grad}(T)$, where $k = \rho c D_E$ is thermal conductivity, and D_E is eddy diffusivity (D_h or D_v).

GFDL MOM version 2 is used for estimation of entropy production in the ocean system. The model domain is a rectangular basin with a cyclic path, representing an idealized Atlantic Ocean. The horizontal grid spacing is 4 degrees. The depth of the ocean is 4500 m with twelve vertical levels. The horizontal and vertical diffusivities (D_h and D_v) are $10^3\ m^2 s^{-1}$ and $10^{-4}\ m^2 s^{-1}$, respectively. We conducted a spin-up experiment under restoring boundary conditions for 5000 years and obtained a steady state with northern sinking circulation. We calculated the global and local entropy productions for the steady state (see Shimokawa & Ozawa (2001) for the results of global entropy production).

3. Local entropy production in the ocean system

Figure 1 shows the distribution of local entropy production for the steady state as stated above. It can be seen from the zonal average of A (Fig. 1(a)) that entropy production is large in shallow layers at low latitudes. This can be seen also in the zonal-depth average of $A \times dV$ (Fig. 1(c)). On the other hand, it can be seen from the depth average of $A \times dV$ (Fig. 1(b)) that entropy production is large at the western boundaries at mid latitudes. Thus, entropy production is highest at low latitudes as the zonal average, but it is greatest at the western boundaries at mid latitudes as the depth average. It can be seen from the Figures of A_x, A_y and A_z (Fig. 1(d), (g) and (j)) that A_x is large in shallow-intermediate layers at mid latitudes, A_y is large in intermediate layers at mid-high latitudes, and A_z is large in shallow layers at low latitudes. It can be also seen from the Figures of $A_x \times dV$, $A_y \times dV$ and $A_z \times dV$ (Fig. 1(e), (f), (h), (i), (k) and (l)) that $A_x \times dV$ is large at the western boundaries at mid latitudes, $A_y \times dV$ is large at mid-high latitudes, and $A_z \times dV$ is large at low latitudes. In addition, it can be seen that the values of A_x ($A_x \times dV$) are smaller than those of A_y ($A_y \times dV$) and A_z ($A_z \times dV$). Thus, there are three regions with large entropy production, namely, shallow layers at low latitudes, western boundaries at mid latitudes, and intermediate layers at high latitudes. It can be assumed that the contribution of shallow layers at low latitudes is due to equatorial upwelling, that of western boundaries at mid latitudes is due to western boundary currents, and that of intermediate layers at high latitudes is due to deep water circulation. It can be also seen that high dissipation regions at high latitudes in the northern hemisphere are reflected in the intermediate layer in the zonal averages of $A \times dV$ and $A_y \times dV$, and the peak of northern hemisphere is larger than that of southern hemisphere in the zonal-depth averages of A and A_y. These features appear to represent the characteristics of the circulation with northern sinking.

Strictly speaking, we should take into account dissipation in a mixed layer and dissipation by convective adjustment for entropy production in the model. The dissipation in a mixed layer can be estimated from the first term in (2) such as

$$B = \frac{\rho c}{T^2} \frac{(T_r - T_s)^2}{\Delta t_r}, \qquad (4)$$

where T_r is restoring temperature (see Shimokawa & Ozawa, 2001 for the distribution), T_s is sea surface temperature in the model, and Δt_r is the relaxation time of 20 days . It is assumed here that $F_h = -k\,\mathrm{grad}(T) = -\rho c D_M\,\mathrm{grad}(T)$, where $k = \rho c D_M$ is thermal conductivity, $D_M = \Delta z_r^2 / \Delta t_r$ is diffusivity in the mixed layer, and Δz_r is a mixed layer thickness of 25 m. The estimated value of B is lower than that of A by three or four orders and is negligible. The dissipation by convective adjustment can be estimated from the first term in (1) such as

$$C = \frac{\rho c}{T_b} \frac{(T_b - T_a)}{\Delta t}, \qquad (5)$$

where T_b is temperature before convective adjustment, T_a is temperature after convective adjustment, and Δt is the time step of 5400 seconds. T_b is identical to T_a at the site where convective adjustment has not occurred. We have confirmed that the value of C is negligible in the steady state of this oceanic general circulation.

References:

S.Shimokawa and H.Ozawa (2001): On thermodynamics of the oceanic general circulation: entropy increase rate of an open dissipative system and its surroundings, Tellus A, 53, 266-277.

S.Shimokawa and H. Ozawa (2002): On thermodynamics of the oceanic general circulation: irreversible transition to a state with higher rate of entropy production, Quarterly Journal of the Royal Meteorological Society, 128, 2115-2128.

S.Shimokawa and H. Ozawa (2005): Thermodynamics of the oceanic general circulation: A global perspective of the ocean system and living systems, In: Non-equilibrium thermodynamics and the production of entropy: Life, Earth, and Beyond (A. Kleidon and R. D. Lorenz eds.), Springer-Verlag (Berlin), 121-134.

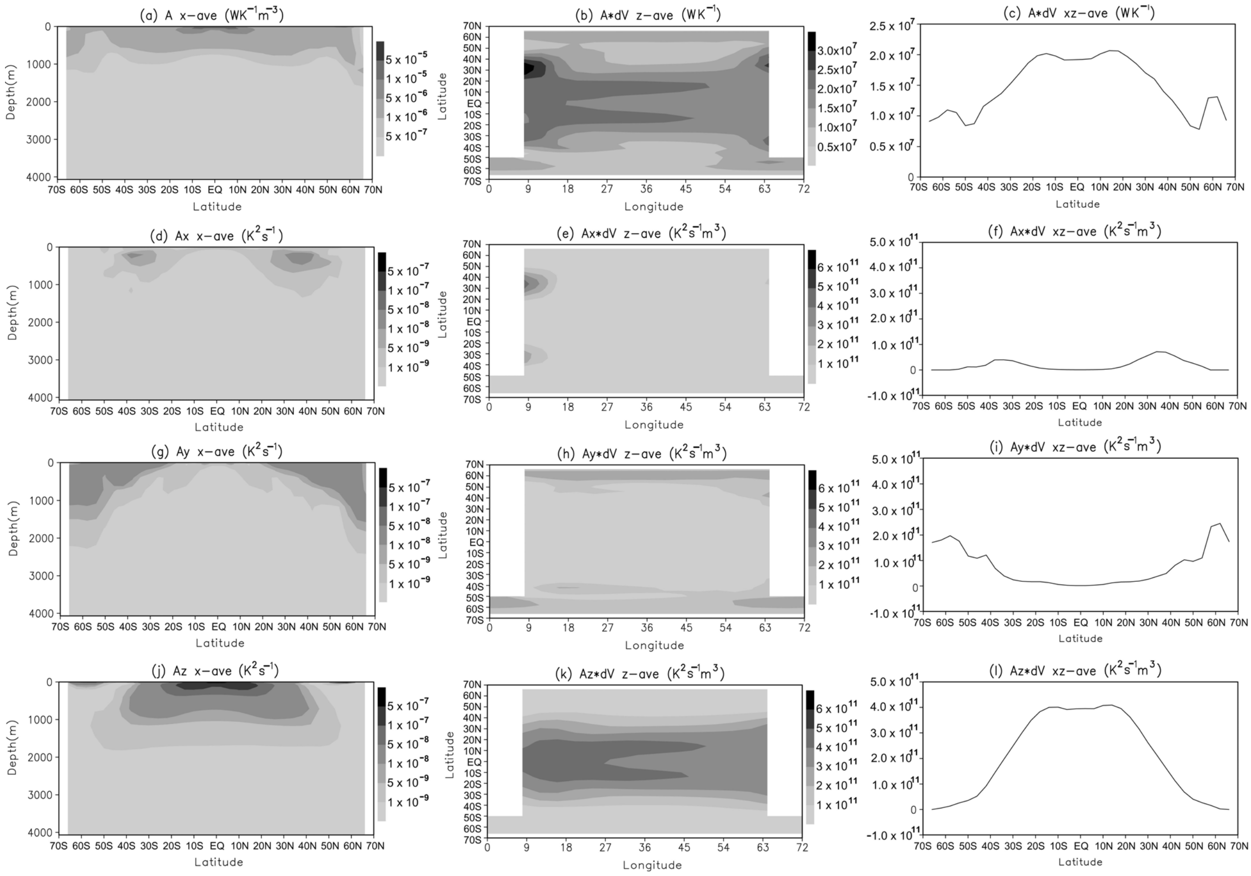

Fig. 1 The distribution of entropy production in the model. (a) zonal average of A, (b) depth average of $A \times dV$, (c) zonal-depth average of $A \times dV$, (d) zonal average of A_x, (e) depth average of $A_x \times dV$, (f) zonal-depth average of $A_x \times dV$, (g) zonal average of A_y, (h) depth average of $A_y \times dV$, (i) zonal-depth average of $A_y \times dV$, (j) zonal average of A_z, (k) depth average of $A_z \times dV$, (l) zonal-depth average of $A_z \times dV$, The unit for A is $W\,K^{-1}\,m^{-3}$. The unit for $A \times dV$ is $W\,K^{-1}$. The unit for A_x, A_y, and A_z is $K^2\,s^{-1}$. The unit for $A_x \times dV$, $A_y \times dV$, and $A_z \times dV$ is $K^2\,s^{-1}m^3$. The quantities not multiplied by dV represent the values at the site, and the quantities multiplied by dV represent the values including the effect of layer thickness.