Semi-Lagrangian advection and conservation[i]

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Semi-Lagrangian (SL) schemes are widely used for the advection component of many modern operational atmospheric models due to their increased efficiency and stability compared to Eulerian schemes. However, a common disadvantage of interpolating SL schemes is the lack of mass and tracer conservation. Though mass conservation may not be critical for short period NWP simulations, it is very important for long period simulations such as those of climate studies. Over a long simulation period, the total mass can drift significantly if no correction is applied. Hence, SL schemes which are inherently mass conserving are desirable. The challenge is to not only achieve inherent conservation, but to do so whilst minimising the additional cost over that of a traditional interpolating SL scheme. This motivated the development of Zerroukat et al. (2002)'s Semi-Lagrangian Inherently Conserving and Efficient (SLICE) algorithm.

There are two ingredients. The first is to rewrite the Eulerian flux form

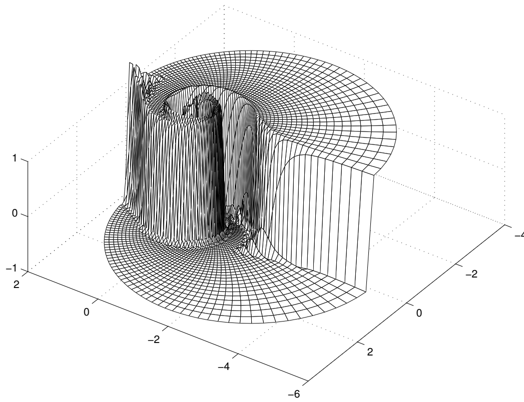
$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0,$$

where ρ is a scalar field transported by velocity $\mathbf{u}$, in a finite-volume Lagrangian form

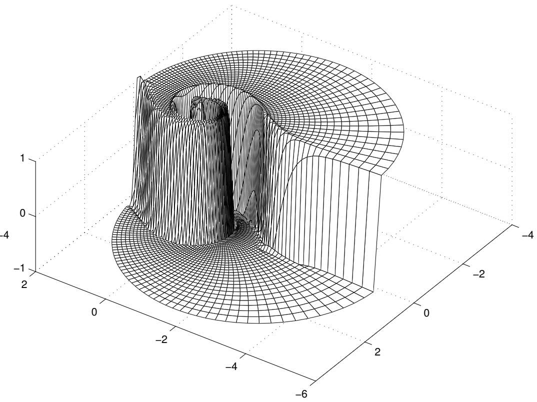
$$\frac{D}{Dt}\int_{\partial V} \rho dV = 0 \Rightarrow M_a^{n+1} = M_d^n.$$

Here ∂V is a fluid parcel or Lagrangian control volume, M_a^{n+1} is its mass at time $(n+1)\,\Delta t$ centred on the arrival location $\mathbf{x}_a$, and M_d^n its mass at time $n\Delta t$ centred on the departure location $\mathbf{x}_d$. The second is to adapt Purser & Leslie (1991)'s cascade remapping strategy to very efficiently decompose a two-dimensional remapping problem (from Eulerian control volumes to Lagrangian ones, or vice-versa) into a number of much-simpler one-dimensional remapping problems - see Zerroukat et al. (2002) for details. An important property of cascade remapping is that it preserves characteristics of the flow, thus minimising splitting errors. Overall, it is found that in addition to exactly conserving mass, the SLICE algorithm is also competitive with standard non-conserving semi-Lagrangian schemes from the viewpoints of both computational efficiency and accuracy.

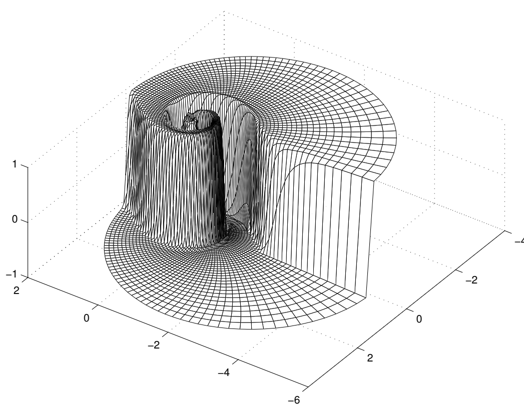
Zerroukat et al. (2002)'s algorithm in planar geometry has been extended to spherical geometry in Zerroukat et al. (2004) with no restriction on Courant numbers. A simple further extension of the SLICE algorithm is described in Zerroukat et al. (2005) which allows monotonicity (and positive-definiteness) to be efficiently imposed in both planar and spherical geometry. This extension operates by first identifying where monotonicity is violated (the detection stage), and by then locally reducing the order of the piecewise polynomial used in the remapping algorithm until monotonicity is regained (the reduction stage). A global minimum and/or a global maximum can similarly be imposed and positive-definiteness is achieved by setting the global minimum to be zero. The resulting monotonicity scheme has been applied to various test cases. Illustrative comparative results for the challenging, non-smooth, deformational problem on the sphere are displayed in Fig. 1.



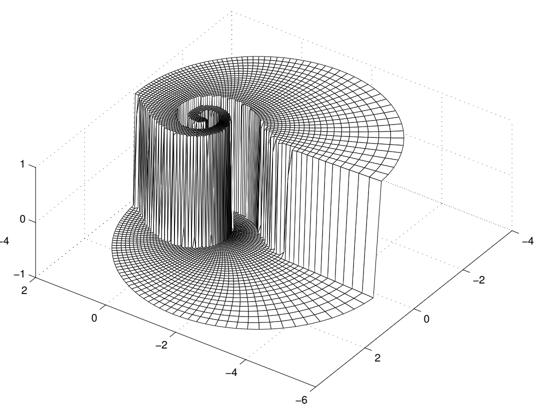
(a) SLICE, *without* monotonicity



(b) bicubic SL, *without* monotonicity



(c) SLICE, *with* monotonicity



(d) analytic

Figure 1: Solutions after 64 timesteps for non smooth deformational flow on a sphere - see Zerroukat et al. (2004) and Zerroukat et al. (2005) for definition of problem and parameters.

References

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