A two-time-level vertically-conservative semi-Lagrangian semi-implicit double Fourier series AGCM

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1. Introduction

We developed a three-time-level vertically conservative semi-Lagrangian scheme (Yoshimura and Matsumura 2003), in which computation of the advection terms is split into the horizontal and the vertical directions and the flux in the vertical direction is evaluated with a conservative semi-Lagrangian scheme.

We have also developed a two-time-level version of the vertically conservative semi-Lagrangian scheme which is about twice more efficient than a three-time-level scheme. We adopt new methods shown below to improve stability of the two-time-level scheme.

We have also succeeded in developing a spectral AGCM using double Fourier series instead of conventional spherical harmonics as basis functions. The double Fourier series model is as accurate as and more efficient than the spherical harmonics model.

2. Improvement of stability of the two-time-level vertically conservative semi-Lagrangian scheme

We improve stability of the two-time-level scheme by adopting SETTLS (Hortal 2002) for nonlinear terms, second-order decentering (Temperton et al. 2001) and the methods shown below to avoid extrapolation in time, a source of instability.

○ The wind integrated in a semi-Lagrangian scheme instead of the wind extrapolated in time is used for horizontal trajectory calculations (Yoshimura 2002). This scheme is similar to but more accurate than the scheme of Gospodinov et al. (2001).

○ The potential temperature instead of the temperature is advected horizontally to avoid calculation and time extrapolation of the heating term related to the horizontal advection. On the other hand, the heating term related to the horizontal divergence is calculated in a conventional finite difference method.

3. Double Fourier series model

T (Temperature), $\ln p_s$, $U = u\cos\theta$ and $V = v\cos\theta$ (where u and v are the component of the horizontal wind and θ is latitude) are expanded from gridpoint space to spectral space with the same basis functions of double Fourier series as in Cheong (2000a, 2000b). For example, T is expanded as

$$T(\lambda,\phi) = \sum_{m=-M}^{M} T_m(\phi) e^{im\lambda}$$

$$T_m(\phi) = \begin{cases} \sum_{n=0}^{N} T_{n,m} \cos n\phi & \text{for } m = 0 \\ \sum_{n=1}^{N} T_{n,m} \sin n\phi & \text{for odd } m \\ \sum_{n=1}^{N'} T_{n,m} \sin\phi \sin n\phi & \text{for even } m(\neq 0), \end{cases}$$

where $\phi = \theta + \pi/2$, θ is latitude and λ is longitude.

The zonal Fourier filter (Cheong 2000a) is used to filter out the high zonal wavenumber components near the poles. The biharmonic spectral filter (Cheong et al. 2002), that is, the 4th-order horizontal diffusion (∇^4) is also used. The same coefficient of the diffusion as in the spherical harmonics model is enough for stable integrations because the use of the semi-Lagrangian scheme improves stability of the double Fourier series model (Yoshimura 2002).

The semi-implicit calculation of the double Fourier series AGCM is efficient. The Helmholtz equation related to the semi-implicit calculation can be easily solved through the diagonalization in the vertical direction (Yessad and Benard 1996) and the inversion of the tridiagonal matrix in the horizontal direction (Cheong 2000b).

4. Prediction experiment

We performed a prediction experiment from 00 UTC 9 July 2002 at TL319L40 resolution. We used three models (a), (b) and (c) shown in Table 1. Fig.1 shows 2-day forecasts of sea level pressure and Fig 2 shows 2-day precipitation forecasts. The results of (a), (b) and (c) are in good agreement. This indicates that these three models are of the same accuracy.

Table 1 also shows the execution time of three-day integrations with 1 CPU of NEC SX6. The execution time of (b) is about half of (a) because the time step of the two-time-level model can be twice of that of the three-time-level model. The execution time of (c) is shorter than (b). This is because the Legendre transform (whose operational count is $O(N^3)$ with N the meridional maximum wavenumber) used in the spherical harmonics model is replaced to the cost-effective Fourier transform (whose operational count is $O(N^2 \log N)$) in the double Fourier series model. The better efficiency of the double Fourier series model against the spherical harmonics model is expected to become more dominant at higher resolutions.

Table 1. The models used in the experiment and the execution time of 3-day integrations

	Integration scheme	Basis function	Resolution	Time step	Execution time
(a)	Three-time-level	Spherical harmonics	TL319L40	10 min.	131 min.
(b)	Two-time-level	Spherical harmonics	TL319L40	20 min.	74 min.
(c)	Two-time-level	Double Fourier series	ML319L40	20 min.	67 min.

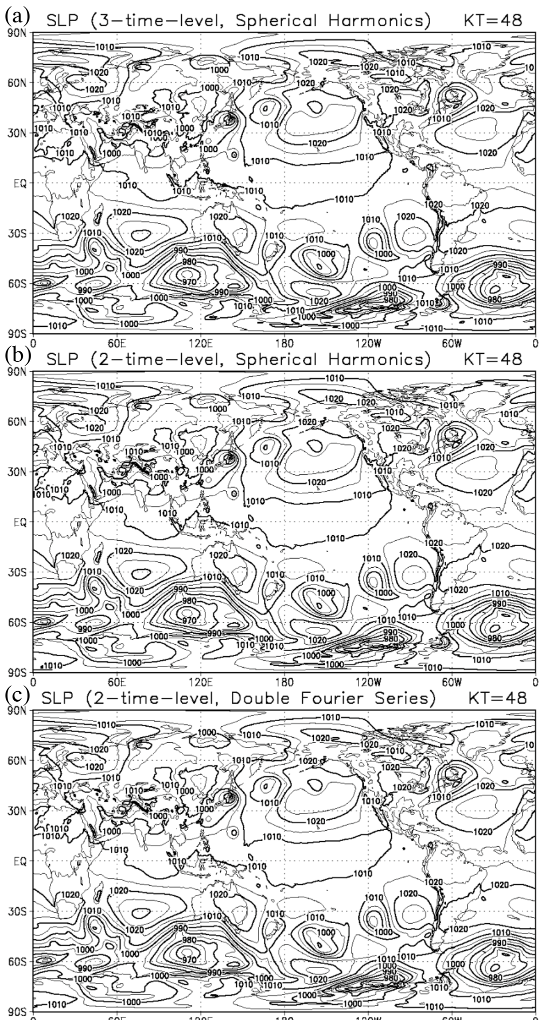

Fig.1. 2-day forecasts of sea level pressure.

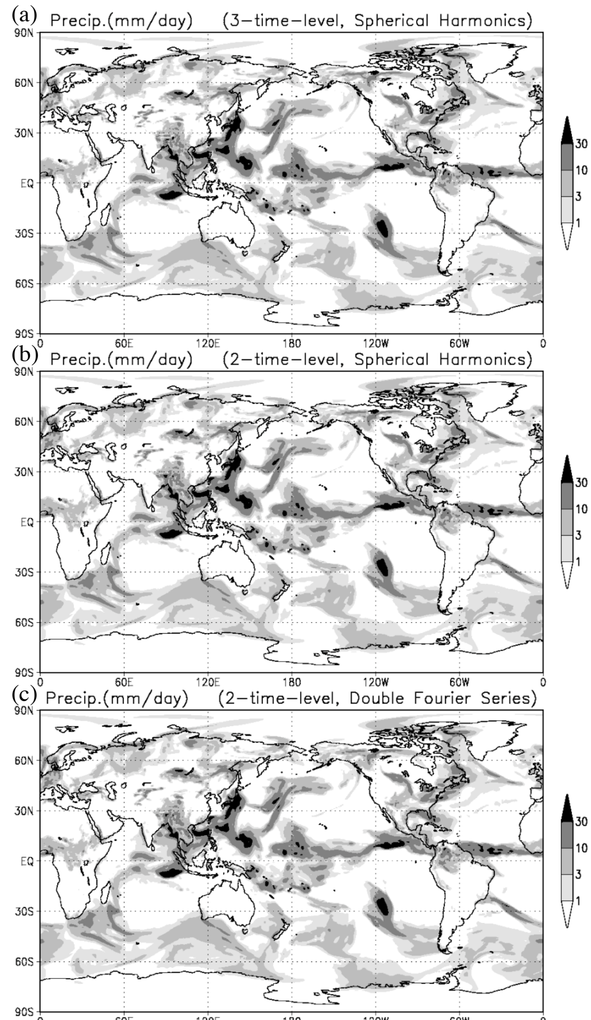

Fig. 2. 2-day precipitation forecasts.

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