Limit cycles of the delayed action oscillator model for ENSO

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Delayed action oscillator (DAO) currently is a wide–accepted model for the El Niño — Southern Oscillation (ENSO) process formation [4]. Here the limit cycles of the DAO model [1] are studied.

According to [3] the dimensionless formulation of the model is

$$dT/dt = \kappa T(t) - T(t-\tau) - [T(t) - rT(t-\tau)]^3 \,, \tag{1}$$

where T — east tropical Pacific sea surface temperature anomaly (from the climatological basic state), t — time, τ — delay, κ and r — constants.

Near–sinusoidal limit cycles with amplitude A and frequency ω can be found semianalytically using [2]. In this case Eq. (1) may be reduced to the transcedential equation for period $P = 2\pi/\omega$

$$\frac{\kappa - \cos\phi}{1 + r^2\,(2 + \cos 2\phi) - r\,(3 + r^2)\cos\phi} = \frac{\sin\phi - \phi/\tau}{r\,[(1 + r^2)\sin\phi - r\sin 2\phi]} \tag{2}$$

($\phi = \omega\tau$). Amplitude can be than found from

$$\frac{3}{4}A^2 = \frac{\kappa - \cos\phi}{1 + r^2\,(2 + \cos 2\phi) - r\,(3 + r^2)\cos\phi}. \tag{3}$$

Example of the solutions for the latter two equations is shown in Fig. 1. Generally these solutions can be found in a wider parameter region than the corresponding values obtained from the numerical integration of Eq. (1) (see, e.g., [3]). This is due to the limit cycle instability with respect to small perturbations in the difference of these two domains. For the stable limit cycles (when both numerically integrated periodic solutions of Eq. (1) and semianalytic solutions exist) the difference between numerically and semianalytically obtained values of amplitude and period is less than 10%.

Larger delay values lead to larger values both of oscillation period and amplitude. Larger values of κ result in larger A while P depends on κ only slightly.

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References

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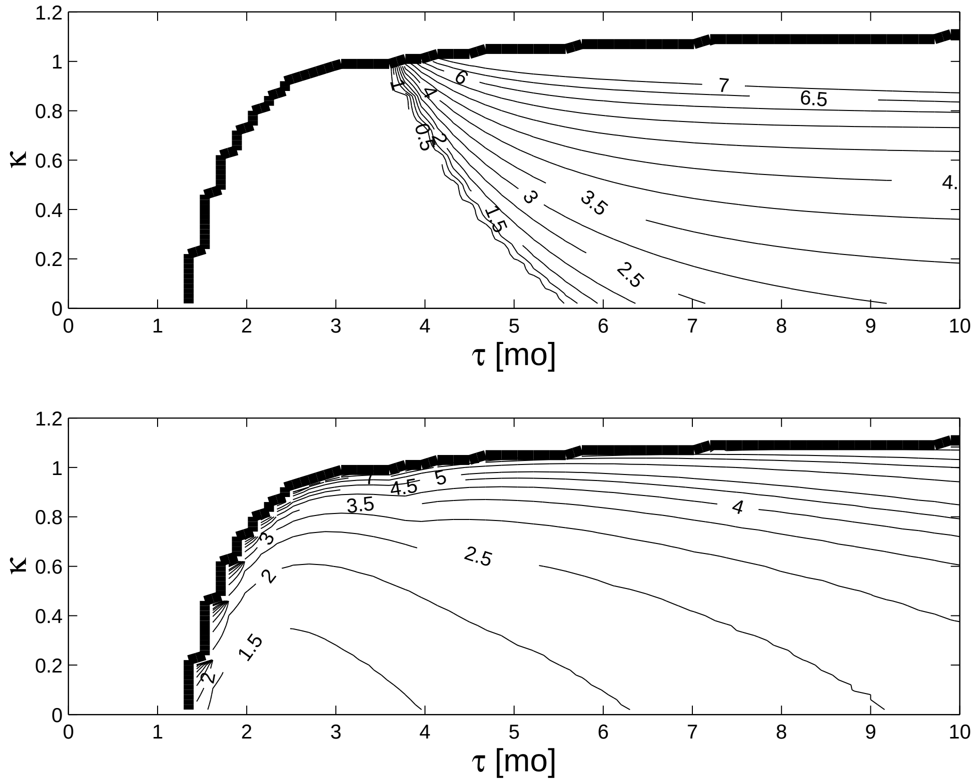

Figure 1: Dimensional amplitude (in Kelvins, upper panel) and period (in years, lower panel) obtained semianalytically for $r = 0.66$ [1]. Dimensional values are computed from the nondimensional ones using temperature and time scales 6.8 K and 3.6 mo respectively. Thick line borders the domain where solutions of Eqs. (2)–(3) exist.