# An Isentropic Model of the Atmosphere

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The isentropic middle atmosphere model of Gregory (1999) has been extended down to the Earth's surface incorporating a new boundary formulation. The model is based on the shallow water model of Thuburn (1997) and predicts PV, divergence $\delta$ and isentropic density $\sigma = \rho \frac{\partial z}{\partial \theta}$ on a hexagonal-icosahedral horizontal grid with an isentropic vertical coordinate. Here we briefly describe the model, in particular the lower boundary formulation, and present some early results.

One of the major difficulties in isentropic modelling of the atmosphere is that the coordinate surfaces intersect the ground. In the past attempts to overcome this problem have been based upon two key techniques: the idea of using extrapolated underground values to calculate finite differences near the ground, and the massless layer approach whereby after hitting the ground an isentropic model layer is extended along the surface with negligible mass.

Our formulation could be considered as a combination of the two. We use a general vertical coordinate $\eta$ which is equal to the potential temperature $\theta$ above ground (see figure 1). When a level hits the ground it retains the same coordinate value $\eta$. Two different density fields are then defined: $\sigma$ is the standard isentropic density above ground and continues underground with non-zero values. $\hat{\sigma}$ is equal to $\sigma$ everywhere above ground and goes to zero at the ground in the same way as in the massless layer method. Initial values for $\sigma$, as well as PV and $\delta$, are extrapolated along isentropes from the surface. Both densities the evolve prognostically according to the mass conservation equation.



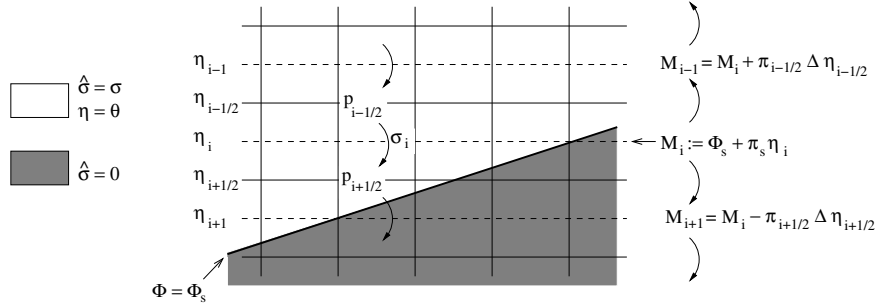Figure 1: A schematic of the lower boundary scheme used in the model. The boundary condition is a specified geopotential $\Phi_s$ and the Montgomery potential $M$ in the lowest massy level is calculated from this as shown for the furthest right column. The arrows indicate directions of integration. $p$ is the pressure, $\pi = c_p \left( \frac{p}{p_0} \right)^\kappa$ and the other symbols are defined in the text.

The real atmosphere is therefore represented by the non-zero $\hat{\sigma}$ region and the underground values of $\sigma$ are there simply as a numerical device used to represent the ground smoothly. This is done in an attempt to avoid another common problem with isentropic models, described by Randall (2000) and summarised here. In isentropic coordinates the horizontal pressure gradient force is the Laplacian of the Montgomery potential $M$ and this is calculated by integrating the pressure up from the surface according to the hydrostatic equation $M_\theta = \pi(p)$. However with discrete isentropic levels intersecting the ground the surface $\theta$ distribution is not smooth leading to noise in $M$ at all levels above an intersection point.
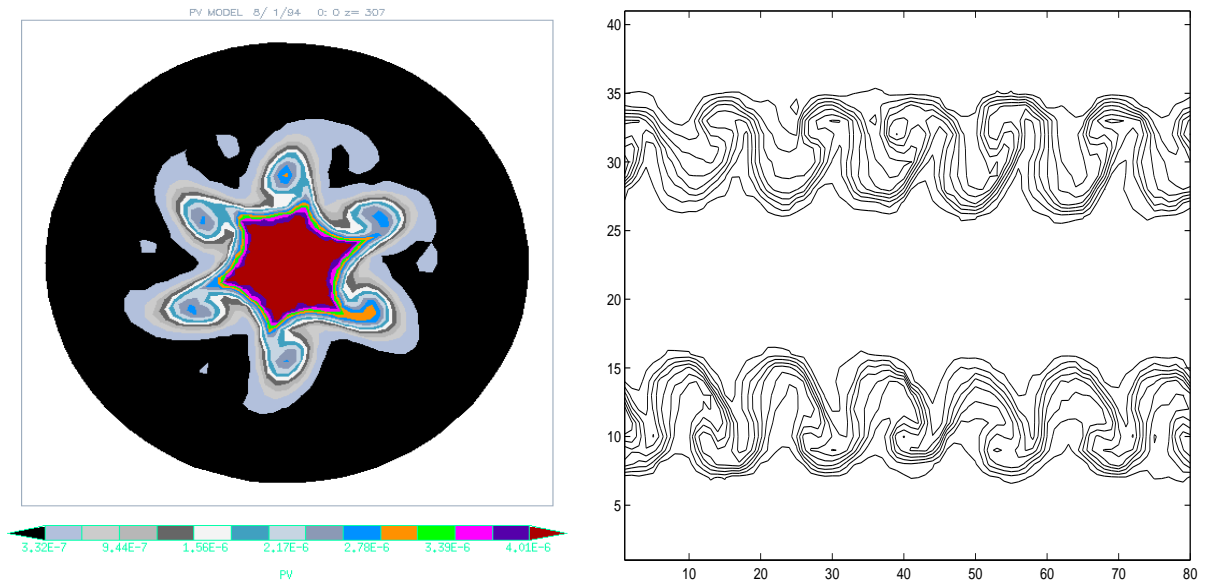
Figure 2: On the left Northern hemisphere PV on the 307K level at day 8 of the baroclinic wave and on the right the global surface $\theta$ at the same time. Horizontal resolution is equivalent to T50.

In our formulation we hope to solve this problem by using the underground $\sigma$ values to interpolate vertically for the exact location of the ground in the $\eta$ coordinate framework so that the surface temperature distribution is smoothly defined. This interpolation is implemented simply by defining $M$ in the lowest massy level using the formula in Figure 1, and $M$ at all levels is then calculated by integrating from here. Note that below ground both this integral and the integral of density to obtain pressure propagate information downwards. No information from below ground propagates upwards to contaminate the real above-ground flow. This however means that in the unphysical below ground region there is no gravity wave feedback mechanism and so the region is unstable. The region has no physical meaning and so the instability is controlled by simply relaxing the flow back towards the initial conditions in such a way that the surface winds are unchanged. The underground region provides for smooth interpolation of surface conditions and is carefully controlled so as to be stable and not to interfere with the real atmospheric flow.

The model has been run successfully in an adiabatic state on simulations such as the development of a baroclinic wave lifecycle as shown in Figure 2. However the model has had difficulty simulating the decay phase of the wave when $\theta$ surfaces become tightly packed at the surface front; in the future it is planned to introduce a simple representaion of diabatic processes in an attempt to improve this.

## References

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