Stablilization of two-time levels semi-implicit algorithms for the Euler system

Pierre Bénard*

* Centre National de Recherches Météorologiques, Météo-France, Toulouse, France

1 Introduction

The Euler equations (EE) system is the most natural system for extending the field of NWP to meso-scales. If the numerical solution of the EE system with three time-levels (3-TL) semi-implicit (SI) schemes is straightforward (e.g. Caya and Laprise, 1999), the use of two time-levels (2-TL) SI schemes leads to severe additional difficulties. For those SI schemes in which the linearization reference thermal field is taken as a constant T^*, it can be shown that the solution of the EE system with a 2-TL SI time-scheme is not possible, whatever height or mass types of coordinates are used (Bénard, 2003, B03 hereafter). This difficulty did not manifest itself so drastically for hydrostatic primitive equation (HPE) systems: although the stability domain was reduced when passing from 3-TL to 2-TL discretizations, a practically viable solution was still possible in 2-TL schemes (Simmons and Temperton, 1997).

However, 2-TL schemes are attractive because of their potentially twice effectiveness compared to 3-TL schemes, hence it is worth trying to understand and circumvent this difficulty. Qian et al. (1998) achieved a stabilization of their 2-TL SI EE system by using a large second-order time-decentering ($\epsilon' = 0.5$ in their notations), but this resulted in an loss of accuracy which is not compatible with NWP. An alternative solution is to use an iterative centred implicit (ICI) scheme instead of a SI scheme, as shown in B03 and in the last issue (2003) of this JSC/CAS WGNE series. However, it must be noted that the benefit in efficiency of the 2-TL scheme disappears due to the cost of the iterative algorithm, unless a significant part of the evolution system is left uniterated (e.g. the physical part). This strategy is used in Côté et al (1998).

In this paper, the causes of the difficulty to solve the EE system with 2-TL SI schemes are discussed, and a solution is proposed to circumvent this problem, without loss of accuracy or effectiveness. The material in the present paper is basically drawn and adapted from Bénard (2004, B04 hereafter).

2 Analysis of the problem and proposed solution

As shown in B04 in a classical academic framework allowing stability analyses, the instability of the 2-TL SI EE system originates from the fact that the thermal (explicitely treated) non-linear residuals corresponding to the terms responsible for horizontally propagating gravity and vertically propagating elastic waves systematically have opposite signs. In other words, if T^* is chosen so as to stabilize horizontally propagating gravity waves, then vertically propagating elastic waves will be unstable, and *vice versa*. The problem was of course not present for HPE systems since they did not allow the propagation of elastic waves.

A natural solution to restore systematically the same sign for thermal non-linear thermal residuals in the terms corresponding to the two types of waves, is thus to introduce different values of T^* for each wave sub-system, that is: T^* for the gravity-wave sub-system, and T_E^* for the vertically propagating elastic-wave sub-system. The stability domains for the first and second sub-systems then become $\overline{T} \leq T^*$ and $\overline{T} \geq T_E^*$ respectively in the analytic framework of B04 (which assumes an isothermal $\overline{T}$ atmosphere). As a consequence, choosing $T_E^* < T^*$ allows a non-empty stability domain to be restored. The stability domain for $\overline{T}$ can thus be arbitrarily extended, by setting T^* arbitrarily warm, and T_E^* arbitrarily cold. However, as shown in Simmons and Temperton, 1997, exaggerating this strategy would finally deteriorate the response of the scheme in terms of accuracy. In practice, T_E^* should thus be chosen colder than the coldest likely temperature, and T^* warmer than the warmest likely temperature. Applying this guideline, no significant loss of accuracy is to be expected, compared to a traditional SI scheme. This proposed modification of the SI scheme is straightforward from any pre-existing application.

3 Practical impact

In order to evaluate the potential benefit of the proposed solution for NWP, the modification was tested in real-case conditions with the adiabatic semi-Lagrangian version of the cooperative NWP Aladin-NH limited-area model, used with a 2-TL SI time-discretization. The model was integrated for 3 hours for a randomly-chosen situation consisting of a strong flow over real topography, in a domain which includes the mountainous Pyrénées region. The horizonal resolution is 2.5 km in horizontal directions, and the time-step is 80 s. The vertical coordinate is the classical mass-based hybrid terrain-following coordinate η, and the domain is discretised along 41 irregular layers with a thickness increasing with height, in the usual NWP fashion. Integrations are performed without any time-filter (see B03 for a discussion on the detrimental effects of time-filters in 2-TL SI EE system).

A weak fourth-order horizontal diffusion is applied to avoid the accumulation of energy in the smallest resolved scales during the course of the integration.

Fig. 1 shows the evolution of the whole domain spectral norms (multiplied by 10^4) of the horizontal vorticity ζ and the divergence D for the traditional and modified versions of the 2-TL SI scheme. The traditional SI scheme is used with $T^* = T_E^*$=300K, and the modified SI scheme with T^*=300 K, T_E^*=150 K. The original 2-TL SI scheme is clearly unstable, since the integrations diverge after 11 time-steps, while the modified 2-TL SI scheme behaves stably during the 3 hours of the integration. This experiment clearly indicates a potential advantage of using the modified SI scheme in NWP with 2-TL EE systems. The advantage of the modification is not limited to this case, and was observed in a wide variety of model configurations, including 2D vertical-plane academic flows, and real cases at higher resolution, with and without physics.

4 Conclusion

The EE system can be solved numerically with a 2-TL SI scheme provided that the residual terms corresponding to gravity and vertically propagating elastic waves are enforced to keep the same sign. This can be achieved by defining two SI reference temperatures T^* and T_E^* instead of one in the classical SI scheme. The proposed modification follows from a more general design of SI schemes, in which the linear reference system is no longer obtained through a linearization of the complete system around a given reference state, as explained in B04. If other non-linear residuals impose the use of an iterative scheme, this modification would allow a more diagonal-dominant implicit operator, and thus, a possibly faster convergence.

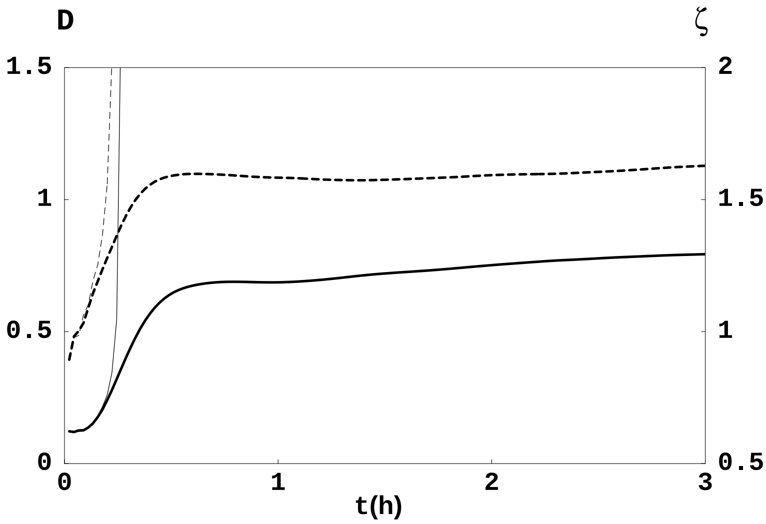

Fig. 1: Evolution of the spectral norm ($\times 10^4$) of vorticity ζ and horizontal divergence D for a real-case with a 2-TL SI EE system. Solid line: vorticity (right axis); dashed line: divergence (left axis); thin line: traditional 2-TL SI scheme; thick line: modified 2-TL SI scheme

References

Bénard, P., 2003: Stability of Semi-Implicit and Iterative Centred-Implicit Time Discretizations for Various Equation Systems Used in NWP. *Mon. Wea. Rev.*, **131**, 2479–2491.

Bénard, P., 2004: On the use of a wider class of linear systems for the design of constant-coefficients semi-implicit time-schemes in NWP. In press at *Mon. Wea. Rev.* (available at the following Internet URL: `http://hal.ccsd.cnrs.fr/view/ccsd-00000881/fr/`)

Caya, D. , and R. Laprise, 1999: A semi-implicit semi-Lagrangian climate model: the Canadian RCM. *Mon. Wea. Rev.*, **127**, 341–362.

Qian, J.-H., F. H. M. Semazzi, and J. S. Scroggs, 1998: A global nonhydrostatic semi-Lagrangian atmospheric model with orography. *Mon. Wea. Rev.*, **126**, 747-771.

Simmons, A. J., and C. Temperton, 1997: Stability of a two-time-level semi-implicit integration scheme for gravity wave motion. *Mon. Wea. Rev.*, **125**, 600-615.