Conservative Semi-Implicit Semi-Lagrangian Method (CSSL) for Transport in Climate and Chemical Models

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Abstract :

The primary objective of this study is to improve the accuracy of transport schemes in coupled chemistry atmosphere models. In the first part of this study, we reproduced the known classical cases of passive advection in Cartesian geometry with both unstaggered (A-Grid) and staggered (C-Grid) grids. A series of 2D validations experiments were performed for 2D passive advection. These experiments have been done using idealized flow with known analytical solutions. Numerical results obtained are found to be in good agreement with analytical solution and those available in literature. In the second part of this study, we solved the full bidimensional equations of the shallow water model with constant Coriolis parameter. These governing equations were discretized by semi-Lagrangian and semi-implicit approach. This work represents the first step towords introducing conservative semi-Lagrangian transport in a shallow-water model.

1. Introduction

The first study concerning the semi-Lagrangian scheme seems to be due to André Robert (1982). Since then, this method has gained a wide acceptance in operational weather forecast and tracer transport models. The main advantages of this approach are its stability and good dispersion proprieties which, when combined with the semi-implicit or fully implicit discretization of the non-advective terms in weather forecast models, yields very robust and efficient yet accurate models. For a complete review of this method and related issues, see Staniforth and Côté (1991). The common problem with the semi-Lagrangian method is the lack of formal conservation of quantities, such as mass, due to interpolation. In fact, the lack of conservation of the semi-Lagrangian method may be a serious problem for climate studies, where many thousands of time steps are needed (long simulations). In applications such as the chemical transport of reacting species by the atmospheric circulation, it is important to ensure that the densities remain positive, and that no spurious sources and sinks are introduced by the transport process, since this could disturb the delicate non-linear chemical balance equations.

Several studies have been devoted to improve this aspect of the method. Such studies can be grouped in two principal categories. The first one concerns shape-preservation, which will guarantee positivity and global conservation either by simple renormalization or with an ingenious combination of low- and high-order interpolations. Both methods have been implemented successfully in semi-Lagrangian schemes. These corrective procedures were developed first by Priestly (1993). His quasi-conservative algorithm consists in restoring the desired quantity whilst minimizing changes to the original solution. The second category concerns the inherently-conservative semi-Lagrangian schemes. It consists in treating the local conservation where the total amount inside a material surface is constant. This type of conservation is relatively expensive and much more difficult to obtain. There is a family of methods similar to the semi-Lagrangian method that enjoys formal conservation at the local level. The most elegant algorithm was developed by Laprise and Plante (1995) for 1D problem. It should be noted that a computationally efficient implementation of these algorithms to higher dimensions is not a simple task. The extension to 3D problem, for example, requires a complex re-mapping which quickly becomes impractical for advective Courant numbers larger than one. Another approach that could be viable in 3D and at large Courant numbers and give local conservation is the use of the so-called cascade interpolation technique in combination with this re-mapping. Rancic (1995) has developed such a scheme for passive advection. His method consists of a natural extension to 2D of a 1D mapping and employs a bi-parabolic piecewise representation. Recently, Zerroukat and al. (2002) developed a Semi-Lagrangian Inherently Conserving and Efficient (SLICE) scheme based on a Control-Volume (CV) approach. The

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algorithm developed by those authors uses multiple sweeps of a 1D conservative remapping algorithm along pre-determined cascade directions and was tested for 2D standard passive advection.

As mentioned above, the major task of this work is to cure the problem of local conservation. In fact, we will be examining the feasibility of optimally combining these ideas for the development of an in-line transport model which will be efficient, conservative, shape-preserving, and applicable to non-hydrostatic models as well. The main purpose of the paper is to describe the principal parts of this investigation realized until now. Validation results of 2D passive advection using the semi-Lagrangian schemes based on the Arakawa C-Grid in Cartesian frame are shown in this paper.

2. The preliminary results: Passive advection

In the first stage, the numerical model was validated for the 2D prognostic equation of passive advection given by:

$$\frac{dF}{dt} = \frac{\partial F}{\partial t} + u\frac{\partial F}{\partial x} + v\frac{\partial F}{\partial y} = 0 \qquad (1)$$

where the function $F(x,y,t)$ stands for any physical variables characterizing the atmosphere.

The first test consists of the classical idealized cyclognesis case defined by an initial circular vortex with a tangential velocity. Only the results will be presented here (see, *Zerroukat et al. (2002) for details). All results have been obtained for the set of parameters:* $\Omega = [0 \quad 10\,m]^2$, $N_x = N_y = 129$, $N_t = 16$, $\Delta t = 0.3125\ [s]$, $V_{\max} = 2.5981\ [m/s]$.

The error measure is defined as the root-mean-square (RMS) difference between the analytical solution and the numerical one with the departure point calculated numerically or analytically.

Table 1 shows the results obtained for the cyclognesis problem. These results are compared with those obtained analytically and available in literature (Zerroukat et al (2002)).

Figure 1 shows the results obtained with the two grids for the classical idealized cyclognesis case described above. These results show the variation of RMS as function of time. We note that the analytical solution obtained by Zerroukat et al. (2002) is completely reproduced.

Table 1: Idealized cyclognesis problem results

		Error : RMS	
Work	**Grid**	**Analytical**	**Numerical**
Zerroukat et al (2002).	A-Grid	0.074217	Not available
	C-Grid	Not available	Not available
Present work	A-Grid	**0.074217**	**0.082840**
	C-Grid	**0.074217**	**0.081407**

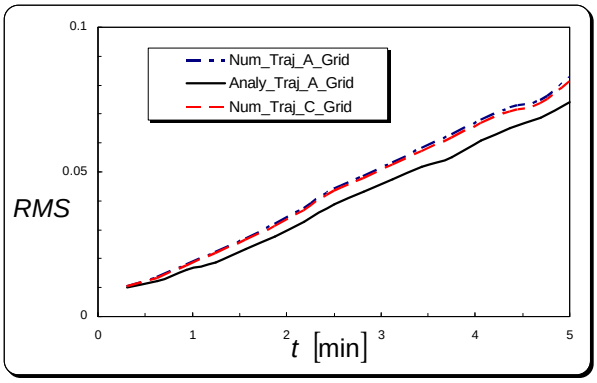

Fig. 1: The variation of root-mean-square, RMS as a function of time t [min]

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