Adaptive algorithm of the suboptimal Kalman filter

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The most fundamental difficulties of the implementation of the Kalman filter theory to the meteorological data assimilation are that it is too computationally expensive and requires too much information [1]. One of the ways to solve this problem is to apply the simplified models in a Kalman filter for calculation of the forecast error covariances (suboptimal Kalman filter).

In the Kalman filter algorithm the forecast error covariance matrix P_k^f is calculated under the formula: $P_k^f = A_{k-1}P_{k-1}^a A_{k-1} + Q_{k-1},$ where A_{k-1} - describes the operator of forecast model, Q_{k-1} is the model error covariance matrix, P_{k-1}^a - the analysis error covariance matrix. In the suboptimal Kalman filter algorithm the on a step of calculation of a matrix P_k^f instead of the operator A_{k-1} the operator of the simplified model $\widetilde{A}_{k-1}$ (smaller dimension) is used. The variants of the simplified models for calculation of a matrix P_k^f are considered in works [2-5].

It is well known, that if Q_{k-1} is not sufficiently well known or if $Q_{k-1} = 0$, the Kalman filter may diverge [1]. In case the covariance matrix of model noise is set zero, there is a fast decrease of theoretical error of the Kalman filter algorithm and, as a consequence, the observations enter on an analysis step with the lesser factors. This effect is named “divergence of the Kalman filter algorithm”. In the given work the adaptive Kalman filter algorithm of estimation of Q_{k-1} is considered. The algorithm allows correct the forecast error covariance matrix P_k^f, calculated with the help of the simplified operator $\widetilde{A}_{k-1}$, also. The algorithm is based on use of vectors of “residuals” (difference between observations and forecast). The forecast of P_k^f with the use of $\widetilde{A}_{k-1}$ we shall consider as the first guess of P_k^f. $\widetilde{A}_{k-1}$ can differ from operator of initial model, as in suboptimal algorithm of the Kalman filter. In adaptive algorithm the diagonal elements of P_k^f are calculated by the method of successive correction. In that method we use the residuals for obtaining the “observed” values of diagonal elements of P_k^f. A full matrix P_k^f we shall restore, considering, that correlations are calculated precisely.

On the fig.1 and fig.2 the dependence of weight coefficients in procedure of the analysis from the space is given. The weights were calculated for the one central point of the region for 0 hour (fig.1) and 12 hour (fig.2). On that figures through w_1 are designated weights calculated on a matrix P_k^f, obtained with the help of adaptive algorithm, w_2 - weights calculated on a matrix

P_k^f , obtained under condition $Q_{k-1} = 0$. So, when we suppose $Q_{k-1} = 0$, the weights become unreal small (Kalman filter diverge), the adaptive algorithm help to avoid the “divergence”.

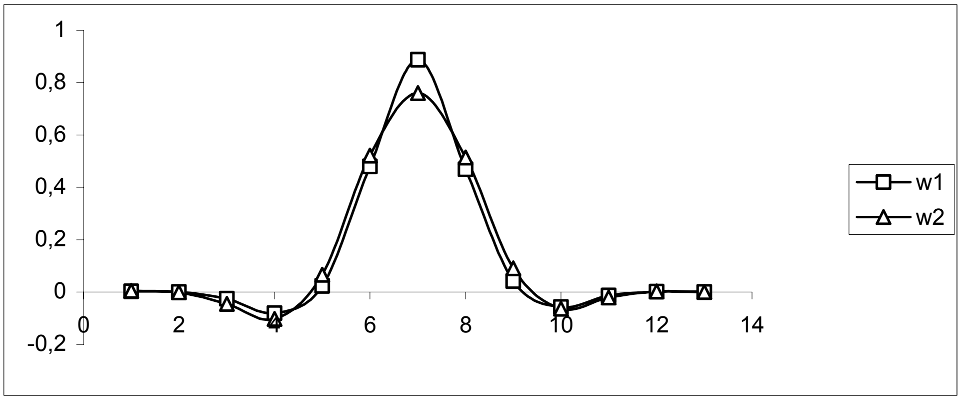

Fig.1

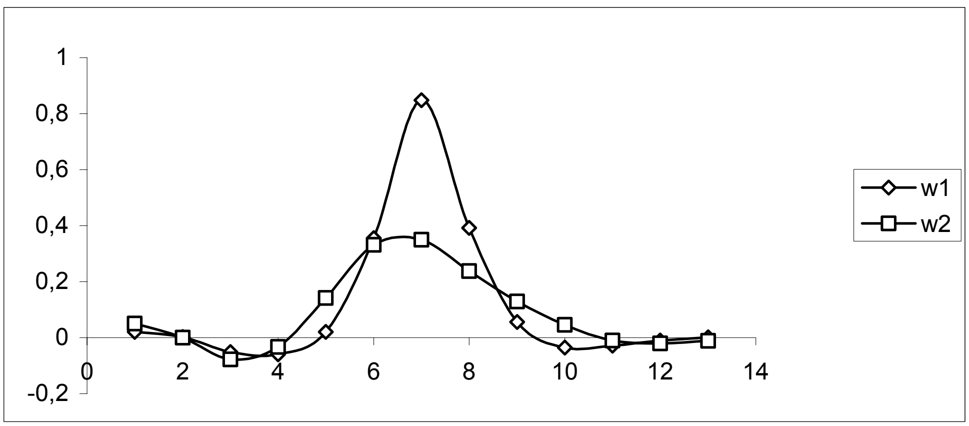

Fig.2

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