A generalized hybrid transformation for tracer advection

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Atmospheric and ocean circulation models often advect variables that vary rapidly in space or have a very large dynamical range. Examples include moisture and chemical species in atmospheric models, and chloroflourocarbons and biogeochemical tracers in ocean models. Such variables present a challenge for numerical advection algorithms, which often produce artificial extrema or "fill in" deep minima. Undesirable side effects include unphysical negative values of chemically active tracers, spurious moisture saturation leading to unphysical precipitation, and an unrealistically shallow stratospheric moisture minimum.

Efforts to overcome these difficulties typically have focused on improving spatial resolution or employing more sophisticated (and inevitably more expensive) advection algorithms. Here we describe a transformed variable approach that enables improvement to be attained without such measures. Under this approach, the quantity that is advected is a transformed variable s that is related to the physical variable q by

$$s = \frac{q_0}{[1 + p\ln(q_0/q)]^{1/p}}, \qquad q < q_0, \tag{1}$$

$$s = q, \qquad q \geq q_0,$$

where q_0 and p are constants. This is a generalization of the hybrid transformation proposed by Boer (1995), for which $p = 1$.

Figure 1 illustrates use of the transformation in a simple one-dimensional advection problem. The advected function has steep gradients and a dynamic range of 10^4, about that of moisture between the troposphere and lower stratosphere. The function has been advected across one cycle of a periodic domain, consisting of 128 grid points. Two advection algorithms are considered. For the numerically diffusive upstream method (left), the transformation improves representation of the minimum and sharp edges. Even more dramatic improvement is realized for spectral advection (left), where, as in some circulation models, modest explicit dissipation has been added to reduce Gibbs fringes.

An obvious hazard is that advection of q becomes non-conservative, even when s is globally conserved. An error analysis indicates that when the leading spatial truncation error is of second order, as for both algorithms considered above, the rate of nonconservation scales as

$$\frac{\partial}{\partial t}\int q(\mathbf{x},t)\, d\mathbf{x} \;\propto\; \int \frac{\partial q}{\partial s}\frac{\partial^2 s}{\partial x^2}\, d\mathbf{x}. \tag{2}$$

When considered as a function of (q_0, p) this rate is generally positive for $q_0 \lesssim \max(q)$ and negative for $q_0 \gtrsim \max(q)$, as in Fig. 2, where the left-hand panel shows (2) evaluated for exact $q(x)$ in Fig. 1, and the right-hand panel accumulated nonconservation for the computation in Fig. 1b. Separating these regimes is a locus of exact global conservation of q; in the Fig. 1 examples, (q_0, p) were chosen to conserve q to within 0.1%. Though the nonconservation rate depends also on the spatial distribution of q, its main features as illustrated in Fig. 2 are not strongly dependent on the function being advected.

Transformation (1) has been tested for spectral advection (with weak dissipation) of moisture and chemical species in the CCCma developmental atmospheric general circulation model AGCM4, with (q_0, p) tuned to maintain near-conservation of these variables. Moisture nonconservation rates as percentages of precipitation, indicated for various (q_0, p) in Fig. 2b taking $\max(q)$=20g/kg, exhibit similar trends to those in the 1d example.

Because of nonconservation issues, the technique described here is not suitable for applications where exact advective conservation of tracers is essential. However, in instances where tracer inventories are established by balances between sources and sinks rather than a memory of initial conditions, the method may provide a useful means for enhancing the fidelity of a given advection algorithm.

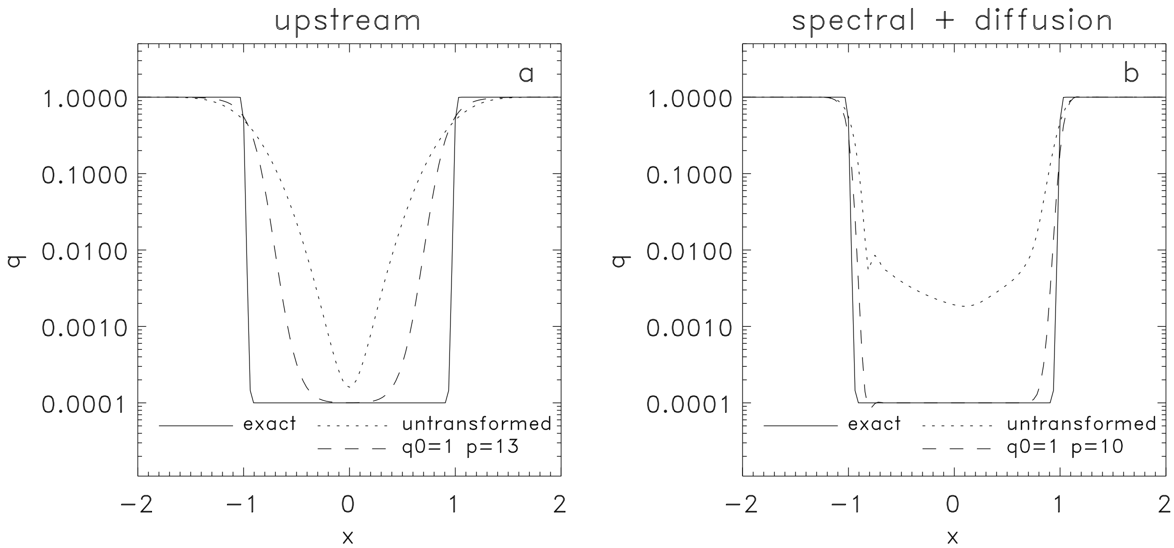

Figure 1: Advection of a 1d function through one cycle of a periodic domain, using transformed and untransformed variables: (a) upstream method; (b) spectral advection with explicit diffusion.

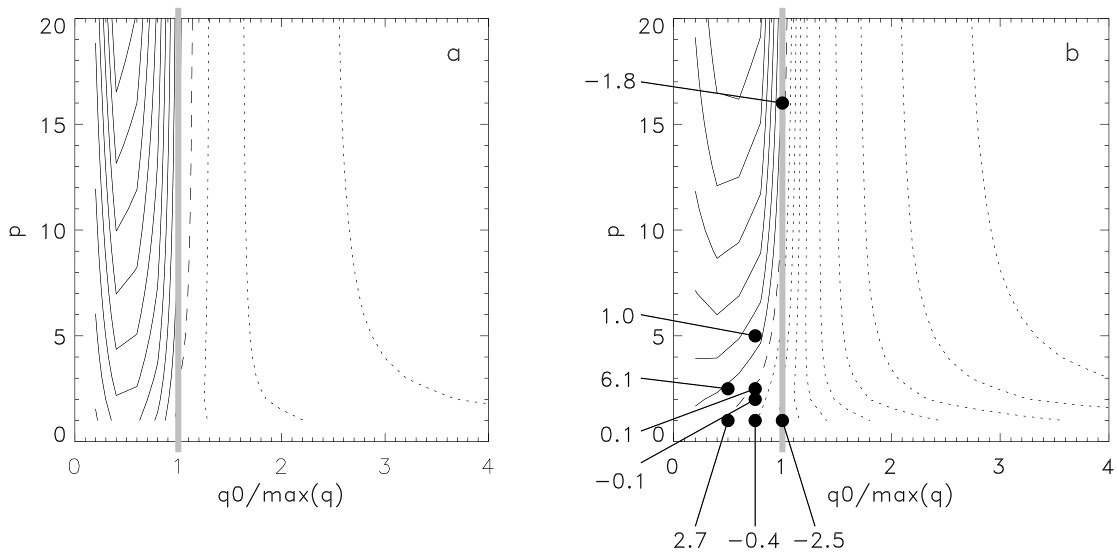

Figure 2: Contours of q nonconservation corresponding to Fig. 1b: (a) as calculated from (2) for exact $q(x)$; (b) as deduced numerically. Solid contours are positive and dotted contours are negative; the zero contour is dashed. Nonconservation rates for moisture in a 3d GCM are indicated in (b).

Reference

Boer, G.J., 1995: A hybrid moisture variable suitable for spectral GCMs. *Research Activities in Atmospheric and Oceanic Modelling. Report No. 21*, WMO/TD-No. 665, World Meteorological Organization, Geneva.