Stable algorithms for the NH version of the LAM "Aladin"

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1 Introduction

In the last issue of this JSC/CAS WGNE report we stated, according to theoretical analyses, that even when chosing "optimal" prognostic nonhydrostatic variables, the classical semi-implicit (SI) scheme suffers from a lack of robustness for Euler Equations in presence of mesoscale features (steep orography, strong nonlinearities,...), and that more sophisticated time-disretisations are required in order to achieve a robustness compatible with the use of semi-Lagrangian (SL) schemes in these circumstances. These previous conclusions are confirmed here by adiabatic numerical experimentation performed with the nonhydrostatic (NH) version of the Aladin limited-area model.

2 ICI schemes

In order to remove the instabilities linked to explicitely treated terms, an Iterative Centred-Implicit (ICI) scheme has been implemented in Aladin-NH in addition to the move from the original (d) prognostic variable to the "optimal" variable (d′) as described in the last issue. The principle of this ICI scheme is to approach the centred-implicit solution by an iterative method. Let the meteorological system to be solved write symbolically as:

$$\frac{\partial \mathcal{X}}{\partial t} = \mathcal{M}.\mathcal{X}. \tag{1}$$

The proposed ICI time-discretisation is defined for the iteration index n as follows: (e.g. for a 2 time-level (2-TL) discretisation):

$$\frac{\mathcal{X}^{+(n)} - \mathcal{X}^0}{\Delta t} = \frac{\mathcal{M}.\mathcal{X}^{+(n-1)} + \mathcal{M}.\mathcal{X}^0}{2} + \frac{\mathcal{L}^*.\mathcal{X}^{+(n)} - \mathcal{L}^*.\mathcal{X}^{+(n-1)}}{2} \tag{2}$$

for $n = 1, 2, ..., N_{\mathrm{iter}}$, where $\mathcal{L}^*$ is a linear operator, and the traditional superscript NWP notation for time levels is adopted ($+$ for time $(t + \Delta t)$, and 0 for time t). In this generalised fixed-point algorithm conditioned by $\mathcal{L}^*$, the initial guess $\mathcal{X}^{+(0)}$ is arbitrary as well as the $\mathcal{L}^*$ operator. However, since $\mathcal{L}^*$ acts as a conditioner, an inappropriate choice of $\mathcal{L}^*$ may prevent the convergence of the algorithm. The choice of $\mathcal{X}^{+(0)}$ is less crucial, but a non-extrapolating $\mathcal{X}^{+(0)} = \mathcal{X}^0$ choice may enhance the stability (see Cullen, 2000). The final $\mathcal{X}^+$ state, valid a $(t + \Delta t)$ is then taken as the last iterated state $\mathcal{X}^{+(N_{\mathrm{iter}})}$ where N_{iter} is an arbitrary number. When the convergence is achieved (which implies $N_{\mathrm{iter}} = \infty$), the obtained scheme is a Centred-Implicit scheme, which can be expected to be very robust.

The analysis of the behaviour of ICI schemes in terms of stability can be performed using similar techniques as in Simmons et al. 1978 for the SI scheme (see e.g. Bénard, 2003). When the scheme is unstable, increasing the number of iterations may increase the growth-rate since the iterative process then becomes divergent. However, the domain of stability of the ICI scheme is generally found wider than for the SI scheme, which justifies its use as an alternative.

In practice, N_{iter} is chosen very small in order to keep an acceptable efficiency. We use the following choices: $\mathcal{X}^{+(0)} = \mathcal{X}^0$, $N_{\mathrm{iter}} = 2$, and $\mathcal{L}^*$ is the same operator as for the classical SI algorithm (defined by $N_{\mathrm{iter}} = 1$). Since only one additional iteration is performed compared to the SI algorithm, this scheme is referred to as "predictor/corrector" (PC) scheme, the SI iteration being the predictor substep and the additional iteration being the corrector one.

3 Results

Extensive testing have been performed to confirm the theoretical statements found by analysis. Two main type of experiments are reported here: academic 2D orographic flows, and adiabatic real-data 3D flow, drawn from the PYREX field-experiment.

For the 2D academic flows, we chose to present a non-hydrostatic non-linear regime characterized by the following settings: smoothly varying nybrid η coordinate, 128 point horizontal extension with Δx= 200m, 100 levels with a regular Δz=300m, Agnesi orography with height h =1000m and half-width a =1000m. The initial thermal profile has a surface temperature T_s=293K, a uniform Brunt-Vaisala frequency N =0.01 s^{-1} up to 12000m, and is isothermal above. The wind is uniform V =10 $\mathrm{m.s}^{-1}$, and the sea-level pressure is π_s=1013.25 hPa. For the definition of the $\mathcal{L}^*$ operator, uniform reference values T^*=220K and π_s^*=900hPa are chosen. A smooth sponge is applied in the stratosphere, and the time-step is Δt=5s for the 3-TL SI SL scheme, and Δt=10s for the 2-TL PC SL (non-extrapolating) scheme. No diffusion or decentering is applied. The stability of the experiments is summarized in Table 1, where the number of completed time-steps is indicated for each configurations.

	3-TL SI	2-TL PC
d	55	20
d'	stable	stable

Table 1: number of completed time-steps for 2D experiments.

It should be noted that the 2-TL PC scheme is more unstable than the 3-TL SI scheme in the first line, reflecting a divergence in the iteration process. The advantage of the PC scheme does not appear explicitely in this table since the move to d′ is sufficient to stabilize the system. However, in other 2D configurations, a clear advantage is observed to the PC scheme (see also 3D case below).

For the 3D experiments, the configuration is as follows: 277×181 horizontal points with Δx=2635m, and 41 levels with a hybrid η coordinate. The initial state is drawn from the PYREX field-experiment data-set, and consists in a strong flow over Pyrénées mountains. A moderate horizontal diffusion (but no decentering) is applied, and the time-step is Δt=15s for the 3-TL SI SL scheme, and Δt=30s for the 2-TL PC SL (non-extrapolating) scheme. The stability of the experiments is summarized in Table 2, where the number of completed time-steps is indicated for each configurations.

	3-TL SI	2-TL PC
d	83	13
d'	281	stable

Table 2: number of completed time-steps for 3D experiments.

This configuration, close to a future operational target, shows a clear advantage to the combination of d′ with a PC scheme.

4 Conclusion

The results are consistent with theoretical statements which were indicating that the classical SI scheme is not robust enough for solving the Euler equations in strongly non-linear regimes, even with an appropriate choice for the set of prognostic variables. The use of more sophisticated ICI schemes allows to remedy to the weaknesses of the SI scheme. It should be noted that the computational cost of the ICI schemes is proportional to N_{iter}, hence this class of scheme has a practical interest only for very low values of N_{iter}. Specifically, 2-TL PC and 3-TL SI schemes have the same computational cost if the time-step of the 2-TL scheme is twice the one of the 3-TL scheme. However, in the case where the physical parameterisation package is computationally expensive (as it will be increasingly the case for mesoscale applications), there is an additional significant advantage to the PC scheme in term of computational cost since most of the physical parameterisations should be called only once per time-step.

References

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