Calculation of the average melting rate of precipitation particles.

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The definition of the average melting rate of precipitation particles is the important problem originating at prediction of such meteorological phenomenon as wet snow, hail, ice-crusted ground etc. The average melting rate E_M depends on temperature profile near the surface, precipitation particles size and their fall velocity. The most common way to define E_M is receiving it as mean from particles mass variance following formula:

$$E_M = \int_0^{\infty} \left(\frac{dm}{dt} \right)_M N(r) dr, \tag{1}$$

where $N(r)$ is the distribution function of precipitation particles. The expression of mass variance of precipitation particle due to melting process has the next form [1]:

$$\left(\frac{dm}{dt} \right)_M = \frac{4\pi r k}{L_f} C(T - T_s), \tag{2}$$

where r - equivalent radius of precipitation particle, L_f - latent heat of melting, k -heat conductivity coefficient of air, T - temperature in the atmosphere, T_s - temperature on the surface of the particle, $C = 1.6 + 0.3 \cdot \mathrm{Re}^{1/2}$, where Re is Reynolds number.

Fall velocity of precipitation particles is calculated by the following formula:

$$U(r) = k_2 \sqrt{r}, \tag{3}$$

where $k_2 = 2 \cdot 10^2$ m$^{1/2}$ /s .

Using the formula (2) for $(dm/dt)_M$ and assumed $N(r)$ as a gamma function with two parameters [2], after solving the integral in (1) we can receive the following formula for E_M :

$$E_M = \frac{3\delta_{PRES}}{\rho_W r_0^2} \frac{k}{L_f} (T - T_s) \left(\frac{c}{(\mu + 2)(\mu + 3)} + d \cdot r_0^{3/4} \frac{\Gamma(\mu + 2.75)}{\Gamma(\mu + 4)} \right) \tag{4}$$

where δ_{PRES} is summary water content (liquid water and ice) of precipitation, r_0 is the mean equivalent radius of precipitation particles, μ is the gamma distribution parameter, assumed as $\mu = 0.41$ accordingly to the work [2], ρ_W is the density of water, $c = 1.6$, $d = 0.3 \cdot (2\rho k_2/\eta)^{1/2} = 1920.0$ m $^{-3/4}$ are the constants.

The results of average melting rate calculation in dependence of different values of temperature in the atmosphere are presented in the left part of Table 1. The calculations were conducted with constant value of summary water content of precipitation $\delta_{PRES} = 0.3$ g/m^3, and different values of mean equivalent radius of precipitation particles - r_0. Results presented in the right part of the table are the values of time $t_M = \delta_{PRES} / E_M$ during which the precipitation particles are completely melted (as a numerator of fraction) and the values of path in the atmosphere $z_M = \overline{U}(r_0) \cdot t_M$ during which melting of precipitation particles occurs (as a denominator of fraction). The values of mean fall velocity of precipitation particles $\overline{U}(r_0)$ and time interval $\Delta\tau$ which needed for precipitation particles to reach the surface from 925 gPa level are presented at the bottom of the table.

The table allow to analyse at what temperatures and mean radius values precipitation can reach the surface as wet snow. The result of comparison of fall time $\Delta\tau$ with melting time t_M under different values of mean radius r_0 shows that precipitation with mean radius $r_0 = 350$ mkm can reach the surface as a wet snow only at temperatures in $1000 \div 925$ gPa level is lower than $1.5^\circ C$. Precipitation with greater mean radius $r_0 = 350$ mkm can reach the surface as a wet snow then the temperatures near the surface is lower than $3^\circ C$. When precipitation has smaller mean radius $r_0 = 200$ mkm particles reach the surface as completely melted at all temperatures near the surface.

The results presented in Table 1 shows that wet snow occurs only at small positive temperatures near the surface and occurrence of this phenomenon depends strongly from values of mean radius of precipitation particles.

Table 1.

Dependence of average melting rate from temperatures near the surface and mean radius values for precipitation which fall out from 925 gPa level.

$t^\circ C$	E_T g/m^{3}c $\cdot 10^2$			t_T min/ z_T km		
	$r_0 = 200$	$r_0 = 350$	$r_0 = 500$	$r_0 = 200$	$r_0 = 350$	$r_0 = 500$
1	0.26	0.12	0.074	$\frac{2.12}{0.32}$	$\frac{4.11}{0.90}$	$\frac{6.75}{1.82}$
2	0.52	0.24	0.15	$\frac{1.06}{0.16}$	$\frac{2.06}{0.45}$	$\frac{3.38}{0.91}$
3	0.78	0.36	0.22	$\frac{0.67}{0.11}$	$\frac{1.46}{0.30}$	$\frac{2.31}{0.61}$
4	1.04	0.48	0.30	$\frac{0.53}{0.08}$	$\frac{1.03}{0.22}$	$\frac{1.69}{0.45}$
5	1.30	0.60	0.37	$\frac{0.40}{0.06}$	$\frac{0.86}{0.18}$	$\frac{1.45}{0.36}$
$\Delta\tau$ min				3.6	2.6	2.3
$\bar{V}$ m/s				2.8	3.6	4.5

References:

1. Mason B. J., 1957: The physics of clouds. Oxford. At the Clarendon press, 542 p.
2. Shlesinger M.E., Oh J.H., Rosenfeld O., 1986: A parameterization of the evaporation of rainfall. Monthly Weather Review, v.116, N 10, pp. 1887-1895.