Time-splitting of gravity waves in the Meteorological Research Institute/Numerical Prediction Division unified Nonhydrostatic Model

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The Japan Meteorological Agency (JMA) started an operational run of a 10 km horizontal resolution mesoscale NWP model in March 2001. The model, MSM, is a high-resolution version of the JMA's operational regional spectral model, where the hydrostatic equilibrium is assumed. As for its initialization, mesoscale 4DVAR is introduced in March 2002. Meanwhile, JMA has a plan to replace MSM by a nonhydrostatic model by the end of FY 2003. Development of an operational nonhydrostatic model for regional NWP (NHM) has been underway, based on the Meteorological Research Institute/Numerical Prediction Division unified nonhydrostatic model (MRI/NPD-NHM; Saito et al., 2001; http://www.mri-jma.go.jp/Dep/fo/mrinpd/INDEXE.htm). Among the three dynamical cores of MRI/NPD-NHM, the Klemp-Wilhelmson type split-explicit time integration scheme (HE-VI scheme) will be used for operation, considering the computational efficiency on the distributed memory parallel computer in the JMA's new NWP system. The HE-VI scheme of MRI/NPD-NHM was incorporated by Muroi et al. (2000), and was based on the formulation of Ikawa (1988). This scheme treats sound waves in the short time step, but has no special treatment for gravity waves. For operational purpose, it is crucial to stabilize the gravity wave modes and remove the dependency of the maximum time step on the atmospheric static stabilities.

The original backward time integrations of the vertical momentum and pressure equations in the HE-VI scheme of MRI/NPD-NHM are written as

$$\frac{W^{\tau+\Delta\tau}-W^{\tau}}{\Delta\tau}+\frac{1}{mG^{\frac{1}{2}}}\frac{\partial P^{\beta}}{\partial z^{*}}+\frac{g}{mC_m{}^{2}}P^{\beta}=\frac{1}{m}BUOY-(ADVW-RW), \quad (1)$$

$$\frac{P^{\tau+\Delta\tau}-P^{\tau}}{\Delta\tau}+C_m{}^{2}(-PFT+m^{2}(\frac{\partial U^{\gamma}}{\partial x}+\frac{\partial V^{\gamma}}{\partial y})$$
$$+m\frac{\partial}{\partial z^{*}}[\frac{1}{G^{\frac{1}{2}}}\{W^{\beta}+m(G^{\frac{1}{2}}G^{13}U^{\gamma}+G^{\frac{1}{2}}G^{23}V^{\gamma})\}]-PRC)=dif.P, \quad (2)$$

where m is the map factor, and terms relating to the tensors for the terrain-following coordinate transformation are represented by $G^{1/2}$, $G^{1/2}G^{13}$ and $G^{1/2}G^{23}$. Terms treated implicitly are indicated with superscripts β and γ. *BUOY* is the buoyancy, *PFT* the thermal expansion of air due to the adiabatic heating, *ADVW* and *RW* are the advection and other residual terms, respectively. *PRC* is the time change of density due to precipitation, and C_m the sound wave speed. To treat the gravity wave modes in the short time step, vertical advection of potential temperature in the basic reference state must be computed in each short time step:

$$\frac{\theta^{\tau+\Delta\tau}-\theta^{\tau}}{\Delta\tau}=-(w^{\tau}\frac{N^{2}\overline{\theta}}{g}+w\frac{\partial\theta'}{\partial z}+u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y})+\frac{Q}{c_p\pi}+dif.\theta, \quad (3)$$

$$\frac{W^{\tau+\Delta\tau}-W^{\tau}}{\Delta\tau}+\frac{1}{mG^{\frac{1}{2}}}\frac{\partial P^{\beta}}{\partial z^{*}}+\frac{g}{mC_m{}^{2}}P^{\beta}=\frac{1}{m}\frac{\rho G^{\frac{1}{2}}\theta_m{}'^{\tau+\Delta\tau}}{\theta_m}-(ADVW-RW), \quad (4)$$

where N is the Brunt-Vaisala's frequency. In case of MRI/NPD-NHM, the advection term for potential temperature is written in the flux form as

$$ADV.\theta = \{m^2(\frac{\partial U\theta}{\partial x} + \frac{\partial V\theta}{\partial y}) + m\frac{\partial W*\theta}{\partial z^*} - \theta DIVT(U,V,W)\}/\rho G^{\frac{1}{2}}, \qquad (5)$$

where $DIVT$ is the total divergence. A flux correction is incorporated as an optional choice to suppress the spurious maxima and minima due to the finite difference and ensure the monotonicity. Besides, computation of the advection term may be further modified in future. In order to utilize the flux form advection Eq. (5), we rewrite Eqs. (3) and (4) as follows:

$$\frac{\theta^{\tau+\Delta\tau} - \theta^{\tau}}{\Delta\tau} = -(w^{\tau}\frac{N^2\overline{\theta}}{g} + w\frac{\partial\theta'}{\partial z} + u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y}) + \frac{Q}{c_p\pi} + dif.\theta$$

$$= -\{\frac{d\overline{\theta}}{dz}(w^{\tau} - w) + ADV\theta\} + \frac{Q}{c_p\pi} + dif.\theta = -\frac{d\overline{\theta}}{dz}(w^{\tau} - w) + \left[\frac{\partial\theta}{\partial t}\right]^*, \qquad (6)$$

$$\frac{W^{\tau+\Delta\tau} - W^{\tau}}{\Delta\tau} + \frac{1}{mG^{\frac{1}{2}}}\frac{\partial P^{\beta}}{\partial z^*} + \frac{g}{mC_m{}^2}P^{\beta}$$

$$= \frac{1}{m}\frac{\rho G^{\frac{1}{2}}\theta^{\tau+\Delta\tau}(1+0.61Q_v)(1-Q_c-Q_r-Q_i-Q_s-Q_g)}{\theta_m}g \; -(ADVW - RW). \quad (7)$$

Here, the second term of *r.h.s.* of Eq. (6) is given by a tentative time integration in the cloud microphysical process, and the first term, which is the vertical advection of the reference state by the difference in w at short and long time steps, is evaluated by the upstream one-sided difference. Since the advection term to compute the tentative time tendency of θ is computed in the flux form, conservation property is kept in the limit where the values of w at short and long time steps coincide.

We also modified the divergence damping filter in NHM to stabilize the sound waves. During the pre-operational test of a 10 km horizontal resolution NHM, no instability for high-frequency modes has been detected using a 40 second long time step increment.

References.

Ikawa, M., 1988: Comparison of some schemes for nonhydrostatic models with orography. *J. Meteor. Soc. Japan*, **66**, 753-776.

Muroi, C., K. Saito, T. Kato and H. Eito, 2000: Development of the MRI/NPD nonhydrostatic model. *CAS/JSC WGNE Research Activities in Atmospheric and Oceanic Modelling*, **30**, 5.25-5.26.

Saito, K., T. Kato, H. Eito and C. Muroi, 2001: Documentation of the Meteorological Research Institute/Numerical Prediction Division unified nonhydrostatic model. *Tech. Rep. MRI*, **42**, 133pp.