Using of the Chebyshev-Laguerre Polynomials for Representing the Vertical Structure of the Atmosphere

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The use of analytical orthogonal functions for describing the horizontal structure of variables in NWP models has been widely used in numerical weather predictions. The vertical structure in such models is usually approximated by parameters specified at discrete levels that are irregular in height. The analytical representation of the vertical structure has not received much attention. An attempt to express the vertical structure in terms of finite sums of analytic orthogonal functions was made by Francis [1]. However, applications of his method were faced with certain difficulties because of the bulky formulas and repeated summations. This paper outlines the possible use of Chebyshev-Laguerre polynomials for representing the vertical atmospheric structure in numerical models.

The Chebyshev-Laguerre polynomials are orthogonal functions defined on the positive half-line and having the weight

$$h(\xi) = \xi^{\beta} e^{-\xi}, \tag{1}$$

where $\beta > -1,\ \xi \in (0, \infty)$.

These polynomials can be represented in terms of the Γ-function as

$$L_n(\xi, \beta) = \sum_{k=0}^{n} \frac{n!\,\Gamma(n+\beta+1)(-\xi)^{n+k}}{\Gamma(k+\beta+1)k!\,(n-k)!}, n \geq 0 \tag{2}$$

and satisfy the orthogonality condition

$$\int_0^{\infty} \xi^{\beta} e^{-\xi} L_q(x, \beta) L_r(\xi, \beta) d\xi = \begin{cases} 0, q \neq r \\ d_q = q!\,\Gamma(q+\beta+1), q = r \end{cases}. \tag{3}$$

Quadrature formulas for Fourier series expansions in terms of the Chebyshev-Laguerre polynomials can be found in [2, 3].

Note two features of the Chebyshev-Laguerre polynomials that can be used in meteorology. The first is that the domain of definition of these polynomials is the interval $(0, \infty)$, which can be interpreted as the thickness of the atmosphere. The second feature is associated with the recurrence relation for the derivative,

$$\frac{\partial L_n(\xi, \beta)}{\partial \xi} = n L_{n-1}(\xi, \beta+1), \tag{4}$$

which can be used in numerical weather forecasting as follows. The geopotential Φ is represented as the Fourier series expansion in terms of the Chebyshev-Laguerre polynomials with parameter β. Then using the equation

$$\frac{\partial}{\partial \xi} \Phi + RT = 0, \tag{5}$$

where $\xi = -ln(p/p_0), p_0 = 1000 hPa,$ we can easily calculate the temperature *T*, because it has the same Fourier series but with the parameter $(\beta + 1)$.

Fourier series expansions of meteorological elements have been applied to the preparation of data for the Russian Hydrometeorological Center spectral atmospheric model. Numerical experiments on the transfer of the geopotential height Φ from *p*-system (17 vertical levels) to σ-system (31 vertical levels) and back were carried out. Thus, the root mean square error was found to decrease down to 10^{-8}hPa. Moreover, the quality of the model outputs was improved, especially at the upper-tropospheric levels and higher. Figure 1 shows the anomaly correlation coefficients for the geopotential height at 100 hPa for the Northern Hemisphere. Curve *1* corresponds to the standard (spline) interpolation method, and curve *2* corresponds to Fourier series expansions in terms of the Chebyshev-Laguerre polynomials.

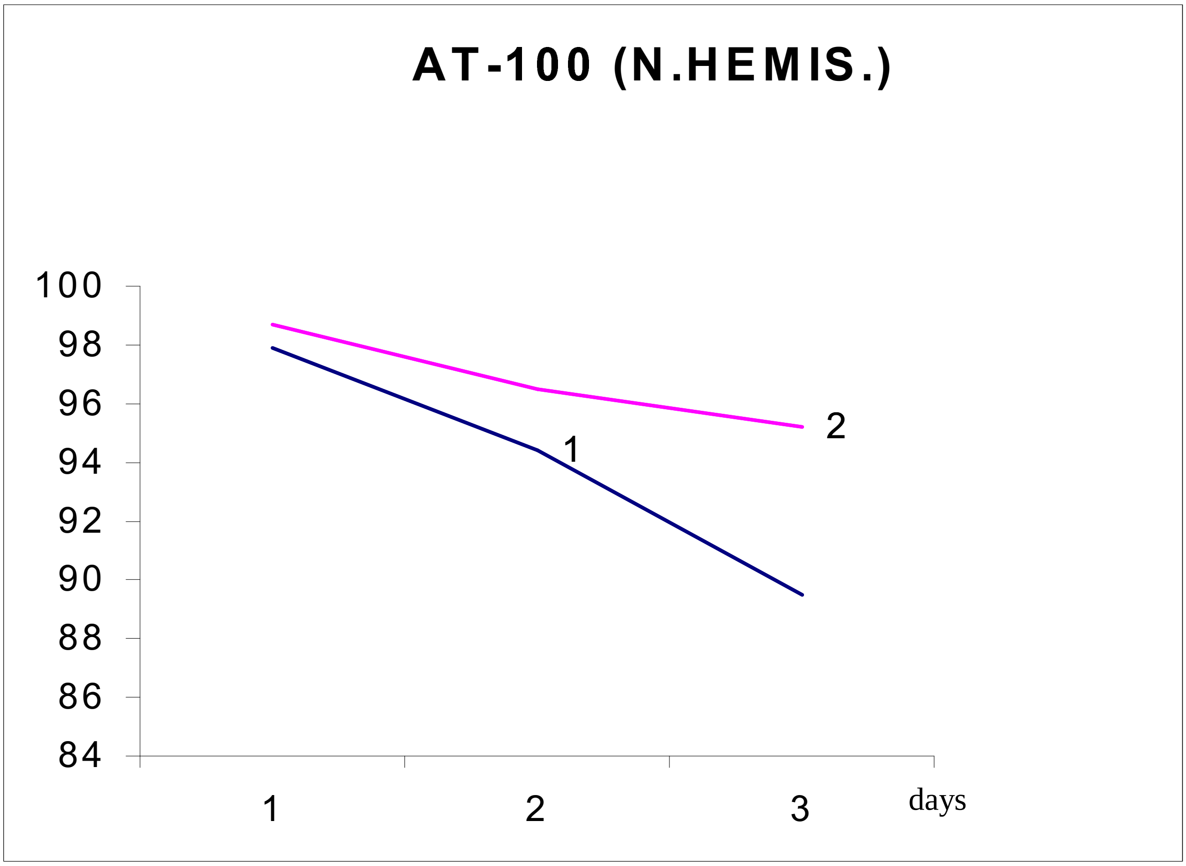

Fig. 1. Anomaly correlation coefficients for the geopotential height at 100 hPa in the case of input data preparation using (1) spline interpolations and (2) Fourier series expansions.

This study was financially supported by the Russian Foundation for Basic Research, grant nos. 00-05-64803, 01-05-65400, and 01-05-65493.

References

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