A model to calculate the covariances of homogeneous isotropic stochastic fields of forecast errors

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Application of the Kalman filter theory to the problem of meteorological data assimilation for the present-day prognostic models is a task whose realization is difficult because of a high order of covariance matrices occurring during this process. At the same time, the atmospheric dynamics is described by particular prognostic equations whose characteristics are well investigated. Besides in the turbulent theory the known analytical equations for structural functions of fields of temperature and wind are received.

A simplified model for calculating the covariance matrices of homogeneous isotropic stochastic fields of forecast errors is proposed in [4]. For a local covariance of forecast errors between two specified points an analytical equation is obtained that is made by analogy with the derivation of equations for the structural functions in the turbulent theory.

Let's name as an error of the forecast a deviation of forecast fields from "true", thus we shall consider, that the errors of the forecast depend only on errors in the initial data. Let "true" state of an atmosphere is described by the baroclinic adiabatic model of atmosphere for a region based on the primitive equations, in (x, y, p) coordinate system [7].

The important feature of numerical algorithm of realization of the given model is the application of idea of G.I. Marchuk of splitting of the dynamic operator of mathematical model on physical processes. In this case at the first stage the system of the equations of advection of mass and temperature along trajectories of motion and on second - system of the equations of adaptation of a wind and geopotential fields are solved.

Let's consider system of the prognostic equations on a time interval (t_k, t_{k+1}). Let $\tilde{u} = u(t_k), \tilde{v} = v(t_k)$.

Let's designate through $\delta u, \delta v, \delta\tau, \delta z$ errors of the forecast of wind field (u, v, τ) and geopotetial z, accordingly. The equations for covariances of the forecast errors of a field δu between two points with coordinates $a_1 = (x_1, y_1, p_1)$ and $a_2 = (x_2, y_2, p_2)$ are obtained by analogy with the derivation given in [6]:

$$\frac{\partial \overline{\delta u_1 \delta u_2}}{\partial t} + \tilde{u}_1 \frac{\partial \overline{\delta u_1 \delta u_2}}{\partial x_1} + \tilde{v}_1 \frac{\partial \overline{\delta u_1 \delta u_2}}{\partial y_1} - f \overline{\delta v_1 \delta u_2} + \tilde{u}_2 \frac{\partial \overline{\delta u_1 \delta u_2}}{\partial x_2} +$$
$$+ \tilde{v}_2 \frac{\partial \overline{\delta u_1 \delta u_2}}{\partial y_2} - f \overline{\delta v_2 \delta u_1} + g \frac{\partial \overline{\delta z_1 \delta u_2}}{\partial x_1} + g \frac{\partial \overline{\delta z_2 \delta u_1}}{\partial x_2} = 0.$$

The equations for covariances of other fields are similar.

From the turbulent theory it is known, that the homogeneous isotropice field of wind velocity in an incompressible fluid does not correlate with any scalar field [6].

Hence, if we assume that stochastic fields of forecast errors are homogeneous and isotropic, the terms with a gradient of geopotential in the equations for covariances are rejected.
If to follow idea of splitting of the dynamic operator of mathematical model on a step of advection on trajectories and step of adaptation of the meteorological fields, at a stage of adaptation in the appropriate equations the terms with a gradient of geopotential thus should be rejected, and so in system of the equations of adaptation there are only terms with the Coriolis force.
The account of Coriolis force means turn of a vector of horizontal wind velocity $U = (u, v)$, and, as in homogeneous isotropic stochastic field all density of distribution by definition do not depend on any turn, those terms with Coriolis force in system of the equations of adaptation should be rejected too. Thus, dynamics of covariances of errors of the forecast, in case they are homogeneous and isotropic, on a small time interval (t_k, t_{k+1}) is described by model of advection of substation on trajectories of particles.
The model developed is used to calculate the covariance of homogeneous isotropic stochastic fields of forecast errors.
Such simplification does not lower essentially order of considered matrices, so the simplified model was obtained under the following additional assumptions:

- the calculation of the forecast error covariances is carred out for the vertical normal modes of the prognistic model;
- the calculation of the forecast error covariances is based on the assumption that the errors of vertical normal modes do not correlate;
- the background fields of velocity components in the advection operator do not depend on vertical coordinate p (i.e., the background flow is close to barotropic).

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